THE STARRY MESSENGER

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Summer 2024
Research
Mentorship
Program

RMP Research Mentorship Program

On The Wave Function Collapse: The Interpretation of Quantum Mechanics - Quantum Mechanics

The Higgs Mechanism - Cosmology

The Search for Magnetic Monopoles - Experimental & Engineering Physics

Explorations in Graph Theory, PageRank & AI - Mathematics

Algebraic Connectivity Influences - Mathematics

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Welcome to the inaugural Issue of "The Starry Messenger", the triannual science magazine of the Global Society of Young Physicists (GSYP). As we embark on this exciting journey with the first edition of The Starry Messenger, I am truly excited to introduce the incredible work of our students from the recently concluded Research Mentorship Programme (RMP) 2024. This issue features five outstanding research reports that delve into the frontiers of modern physics:

1. **On The Wave Function Collapse: The Interpretations of Quantum Mechanics**
2. **The Higgs Mechanism**
3. **The Search for Magnetic Monopoles**
4. **Explorations in Graph Theory, PageRank & AI**
5. **Algebraic Connectivity Influences**

The Global Society of Young Physicists (GSYP) was founded with a vision to connect young physics enthusiasts worldwide, fostering a community dedicated to advancing physics education. GSYP is a registered community interest company that supports budding physicists through a range of initiatives, including the Research Mentorship Programme (RMP), where students are paired with experienced mentors to undertake challenging research projects.

After five intense weeks, we concluded the RMP 2024, marking an incredible journey for all involved. As the Chief Executive Officer of GSYP and co-creator and lead coordinator of this programme, I had the privilege of guiding our brilliant students through their research projects. Serving as a mentor for the Quantum Mechanics course, I led a group of five passionate and brilliant students through five weeks of intensive lectures and research.

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Throughout the programme, our students demonstrated exceptional dedication and curiosity, exploring complex topics such as Schrödinger's equation, quantum tunnelling, Bell's inequality, and even black hole information theory. Their efforts have culminated in remarkable research, just as the other teams.

The foundation of GSYP and the RMP was established through a personal experience that fueled my passion for research and education. Years ago, at a summer school on Quantum Cryptography at the University of Waterloo, I met like-minded individuals, including my dear friend and colleague Leo Nagel, who is the Senior Advisor at GSYP. Leo and I took the initial steps in founding GSYP, while embarking on a research project titled "Measurement of Thermal Conductivity of Thin and Thick Layer Graphene via Phonon Dispersion by Inelastic X-Ray Collision Produced Through Bremsstrahlung Radiation." As a team we had to work on this project without a mentor. The experience of working without a mentor underscored the importance of guided support for students undertaking research projects for me, laying the groundwork for what would become GSYP and the RMP.

This experience then inspired the creation of GSYP and the RMP, which aims to provide young physics enthusiasts with the mentorship and resources we lacked. Today, I am immensely proud of the outstanding work our students and mentors have accomplished. I would like to extend my thanks to the distinguished mentors of RMP 2024, and now fellows of GSYP: Toby, Piyamni, Duc and Ethan. I would also like to thank our Head of Marketing, Chiara Allegri. Last but not least, I would like to thank our students, now RMP Alumni, who made all of this possible. Thank you for all your hard work.

As always, we as GSYP encourage you to support and join our mission to further physics education and connect young physics enthusiasts worldwide. For more information, please refer to our posters at the end of our magazine.

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Become a Member of GSYP!

As a member of GSYP, you gain access to exclusive benefits that will enhance your journey as a young physicist. Members can request reviews from research mentors who are conducting cutting-edge studies in Physics. After a thorough editorial review, you can submit your work for publication in our science magazine. Additionally, you will have the opportunity to attend talks by renowned science experts brought in by GSYP.

Membership is currently free, so join us and take advantage of these incredible opportunities.

Science Magazine Schedule

Our Science Magazine will be published three times a year:

- **April**: Independent research published before the annual RMP.
- **August**: Research reports from the annual RMP.
- **December**: Independent research published after the annual RMP.

Stay tuned for our next issue in December 2024, where we will feature more research reports and provide details on the upcoming RMP 2025!

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https://youtu.be/4IVJRYkERAQ?si=8u_q1XyCgonkdaVv

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We aim for sound science but not editorial orthodoxy.

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“On The Wave Function Collapse: The Interpretations of Quantum Mechanics”

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Gonullu & Howells, Peneda, Rossi, Shahzad

Abstract

The aim of this research report is to give an overview of some interpretations of quantum mechanics, and how they portray the collapse of the wavefunction. The ultimate goal for quantum physicists is to discover which interpretation describes the world in the most complete way possible, but this report does not seek to label any interpretations as correct or incorrect, only to highlight what each interpretation is and its theoretical limitations. The Copenhagen Interpretation is the most widely accepted, but is challenged by other interpretations, especially in how they explain the wave function collapse. The Many-Worlds Interpretation avoids contradictions in multi-observer scenarios by disregarding wave function collapse, but it struggles to explain specific outcomes, while the CSL model offers a smoother explanation for particle behaviour during measurements.

1. Introduction

This research report discusses various interpretations of quantum mechanics, focusing on the Copenhagen Interpretation, Many-Worlds theory, the Transactional Interpretation, and the Continuous Spontaneous Localization (CSL) model. The various interpretations of quantum mechanics are regarded as different mathematical theories that might be able to explain the reality of what is observed. Each interpretation is explored in terms of its approach to the wave function collapse. In quantum mechanics, particles like electrons and photons can exist in a superposition, meaning they can be in multiple states or places at the same time. However, when a measurement is made, they are observed in just one place. This transition from multiple possible states to a single defined outcome is called the collapse of the wavefunction.

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The Copenhagen Interpretation

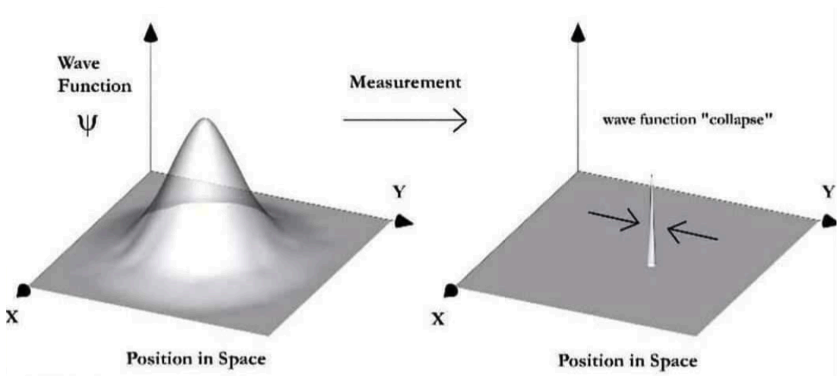

Figure 1. Shows a visual representation of the collapse of the wavefunction upon measurement.

The Copenhagen interpretation is the first interpretation ever developed of Quantum Mechanics. Developed mainly by Niels Bohr and Werner Heisenberg in the 1920s, it is still the most widely taught interpretation (Bassi & Ghirardi, n.d.). The interpretation is built on the idea that particles, such as electrons and photons, exhibit both wave-like behaviours and particle-like behaviours. This is called the wave-particle duality and it is described by the wavefunction ψ (Bassi & Ghirardi, n.d.). The wave function evolves according to Schrodinger's equation (1.1)

$$i\hbar \frac{\partial}{\partial t}\Psi(s,t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(s,t) + V(s,t)\psi(s,t) \qquad (1.1)$$

Where $\hbar$ is Plank's constant divided by 2π; $\Psi(s,t)$ is the wave function defined over space (s) and time (t); m is the mass of the particle; $\nabla^2$is the Laplacian operator $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$, and V(s,t) is the potential energy influencing the particle.

The wave function indicates the probabilities of finding a particle in various states.

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According to the Copenhagen interpretation, the act of measuring causes the wavefunction to collapse to a single eigenstate, corresponding to the measured value. (Ghirardi et al., 1986). If a system is described by a wavefunction, ψ, and an observable $\hat{A}$, with eigenstates ϕ_n, then when a measurement is made the probability P of an outcome a_n is given by the equation

$$P(a_n) = |\langle \phi_n | \psi \rangle|^2 \tag{1.3}$$

Equation (1.3) represents the Born rule, which states that the probability of finding a particle in a particular state is given by the square of the amplitude of the wavefunction. Intuitively, it becomes obvious that the process is inherently probabilistic.

The Copenhagen interpretation also introduces two important principles. One is complementarity, which states that both the particle and the wave aspects of quantum objects are complementary (Bassi & Ghirardi, n.d.). To have a full description of quantum phenomena, both aspects are needed, but they cannot be observed simultaneously. The other is Heisenberg's uncertainty principle, which states that the product of the uncertainty of momentum and uncertainty in position of a particle has to be greater or equal to Plank's constant h divided by 4π (Bassi & Ghirardi, n.d.). For example: the more precisely momentum is measured, the less precisely the position of the particle can be known. This is mathematically described in (1.4)

$$\Delta x \Delta p \geq \frac{h}{4\pi} \tag{1.4}$$

Where Δx is the uncertainty of the position, and Δp is the uncertainty in momentum.

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The Many-Worlds Theory

In this theory, the superposition of the system's states is described by a combined wave function. Each superposition element is represented by a "branch" (which would then be referred to as "world", term coined by DeWitt in the 1970s), therefore, different outcomes are represented by different "branches" (as shown in figure 2.); different outcomes are observed and experienced in different "branches" by the observer. Since the quantum dynamical evolution of the wave function (according to the Schrodinger equation) is thought to be linear, each world is unfolded independently, with the different superposed states being respected (as shown in Figure 2).

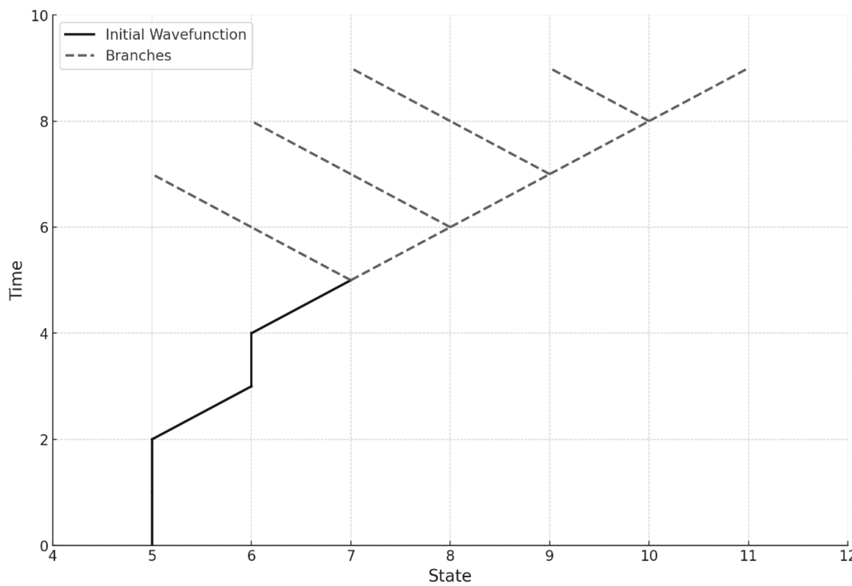

Figure 2. A wave function 'branching out', rather than collapsing. It is shown how the evolution in time of the wave function is linear in describing the different possible outcomes of different quantum events.

The observer-system relation as a composite quantum system

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The superposition of two quantum states is caused by the linear combination of two quantum states linked to two different quantum subsystems. As pairs of subsystems' quantum states are superposed, the superposition is described by the following wave function:

$$\psi = \sum_{i,j} \alpha_{ij} \varphi^{i}\left(x_{1}\right) \eta^{j}\left(x_{2}\right) \tag{2.1}$$

Where the sum of the different possible states (*i* and *j*) of the subsystems is defined by $\sum_{i,j}$, the amplitude and phase of each subsystem's wave function in the superposition is weighted by the coefficients α_{ij}, the *i*-th state of the first subsystem is defined by the function $\varphi^{i}\left(x_{1}\right)$ (with x_1 being the spatial coordinate related to the first subsystem) and the *j*-th state of the second subsystem is defined by the function $\eta^{j}\left(x_{2}\right)$ (with x_2 being the spatial coordinate related to the second subsystem).

In this case, the system is not described by the single states of the subsystems x_1 and x_2, instead, it is only described by the superposition of their states *i* and *j*, therefore, it can be deduced that the two subsystems are entangled.

This wave function specifically can be used in the description of a composite system, with the observer being represented by one subsystem and the observed system by the other: indeed, in the Everettian interpretation, observers are regarded as usual quantum systems. The measurement process is regarded as an interaction between two quantum subsystems, meaning that the property of the measured subsystem is entangled to a quantity in the measuring subsystem. This allows for a consistent picture between the appearance of phenomena and the usual probabilistic interpretation of quantum mechanics.

In Everett's interpretation: the states of a quantum system are represented as vectors in a Hilbert space, a complex vector space with the product of two quantum

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states bra <Ψ| and ket |Φ>, and the time evolution of an isolated quantum system is described by a linear wave function (2.2) (which is referred to as the "universal wave function") containing the description for different world wave functions and making it mathematically comparable to the wave function for the superposition of subsystem states (2.1):

$$\psi = \sum_{i,j} \alpha_{ij}\varphi^{i}\left(x_1\right)\eta^{j}\left(x_2\right) \qquad (2.1)$$

$$\psi_{universe} = \sum_{i,j} \alpha_{ij}\psi_{world\ ij} \qquad (2.2)$$

The Transactional Interpretation

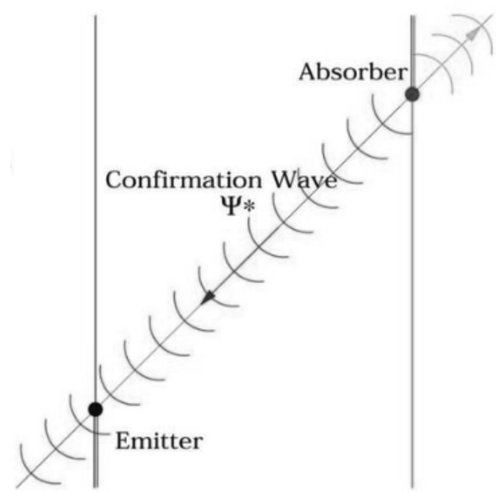

Figure 3. Shows how retarded and advanced waves are exchanged in a completed quantum event.

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Another interpretation of quantum mechanics is the Transactional Interpretation: completed quantum events are described as exchanges of advanced and retarded waves, which are the transactions, known as 'handshakes'. It is noted that this interpretation is explicitly nonlocal but also relativistically invariant and causal. Furthermore, the Heisenberg uncertainty principle and Born probability law are naturally justified by this interpretation. The probability of a transaction is given by: $P = |\langle \Psi f | \Psi i \rangle|^2$ where Ψi is the initial state and Ψf is the final state.

In the transactional interpretation, wave functions are allowed to be interpreted as physical waves. Therefore the wave function Ψ(x,t) is represented by a retarded (offer) wave moving forward in time; the wave function Ψ*(x,t) is represented by an advanced (confirmation) wave moving backward in time. When considering electromagnetic interactions, these wave functions are solutions to the wave equation:

$$(\hbar c)^2 \nabla^2 \psi = \frac{\hbar^2 \partial^2 \psi}{\partial t^2} \qquad (3.1)$$

A problem is posed because no advanced solutions are provided by Schrödinger's equation (which is fundamental to quantum mechanics), and thus the entire interpretation appears to fall apart. However, Schrödinger's equation is not physically correct because it is not relativistically invariant. When a relativistic wave equation is taken into account (as by Bjorke and Drell), the reduction procedure leads to two distinct equations.

The time-dependent Schrödinger equation (having no advanced solutions) is:

$$i\hbar \, \partial / \partial t \cdot \Psi(x,t) = H\Psi(x,t) \qquad (3.2)$$

The advanced wave is described by its complex conjugate (having only advanced solutions):

$$-i\hbar \, \partial / \partial t \cdot \Psi^*(x,t) = H\Psi^*(x,t) \qquad (3.3)$$

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Additionally, one part of quantum mechanics is quantum electrodynamics, and the transactional interpretation of electrodynamics is the Wheeler-Feynman approach. This is a time-symmetric theory that is presented to solve the problem of self-interaction in electrodynamics. In the theory, absorbers (charged particles in the future that respond to emitted radiation) generate waves which are sent backwards in time to the source. Emitters simultaneously emit radiation. By including both advanced and retarded waves in the Wheeler-Feynman approach, the problem of self-energy is eliminated. The retarded wave is noted to have positive eigenvalues for its properties like energy and momentum, and the advanced wave is noted to have negative eigenvalues. Thus, the two waves are effectively cancelled out. It is noted that this theory is an interpretation of electrodynamics and is not widely accepted, but it yields identical results to conventional electrodynamics. (Cramer, 1986).

The Continuous Spontaneous Localization (CSL) model

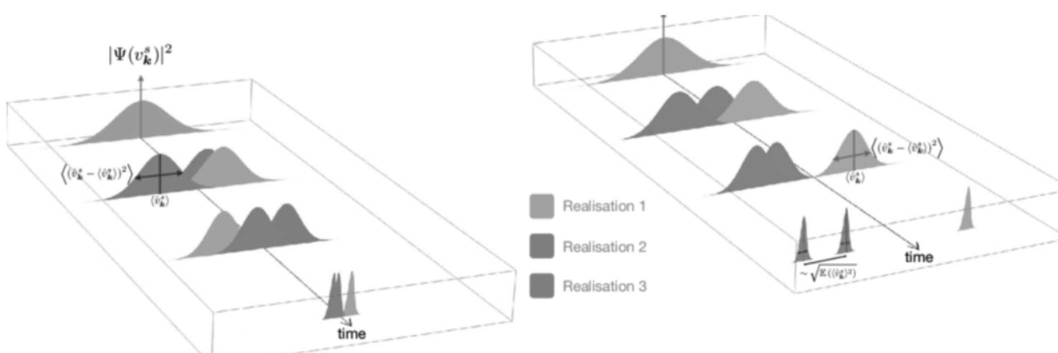

Figure 4. The image on the left is the stochastic trajectories of each wave function, where the criteria of CSL is not satisfied. The right image shows when different realisations correspond to separated outcomes, where the criteria of CSL is satisfied.

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There are different theories about how a wave function collapse happens. One theory is the GRW model, which suggests that particles randomly collapse to one state at specific times, but this model has some limitations. The first limitation is that collapse happens at random, discrete times. The second is that it does not work well for particles that are identical (Ghirardi et al., 1986).

The CSL model improves on the GRW model by making the collapse happen continuously, not just at random times. This means that instead of the particle suddenly collapsing at a state, it gradually settles into one state over time (Pearle, 1989). The main particular feature that allows this model to work is the constant interaction between the particle and the noisefield. This field causes the particle to slowly collapse into one state. This results in the continuous process of, instead of collapsing suddenly, the particle's state changes continuously because of this interaction (Diosi, 1989).

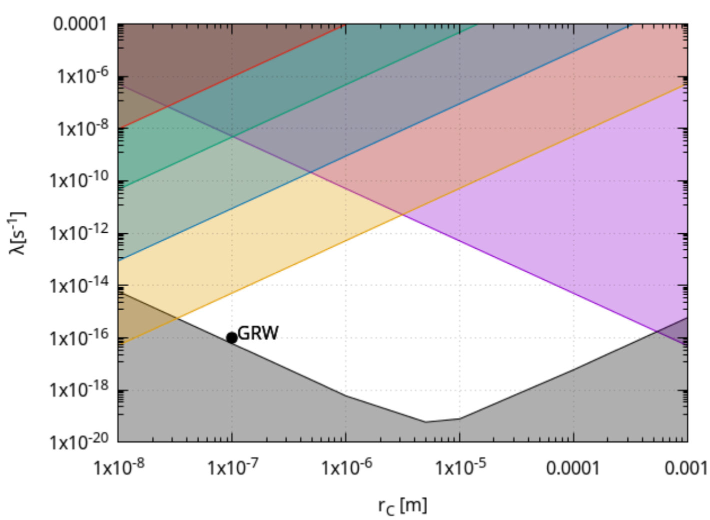

Figure 5. Schematic exclusion plot of the CSL parameters. The grey area represents the theoretical lower bound for macroscopic behaviour. The other shaded areas represent other exclusion areas of the CSL model, which are not significant for the discussion in this report.

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The CSL model modifies the standard Schrödinger equation by adding nonlinear and stochastic terms. The evolution of the state vector$|\Psi(t)\rangle|\Psi(t)\rangle$ is governed by equation (4.1).

$$d|\psi(t) = \left[-\frac{i}{\hbar}\hat{H}dt + \sqrt{\lambda}\sum_j\left(\hat{A}_j - \langle\hat{A}_j\rangle_t\right)dW_j(t) - \frac{\lambda}{2}\sum_j\left(\hat{A}_j - \langle\hat{A}_j\rangle_t\right)^2 dt \right]|\psi(t)\rangle \quad (4.1)$$

Where $\hat{H}$ is the Hamiltonian of the system, $\sqrt{\lambda}\sum_j\left(\hat{A}_j - \langle\hat{A}_j\rangle_t\right)$ is the collapse inducting operator, $dW_j(t)$ is the stochastic noise term, and $\frac{\lambda}{2}\sum_j\left(\hat{A}_j - \langle\hat{A}_j\rangle_t\right)^2$ is the diffusion rate parameter. This modification of the Schrödinger is what allows for the wavefunction to collapse spontaneously and continuously (Bassi & Ghirardi, n.d.).

A particle that has spin up or down, can represent how the model works. Initially, it might be in a superposition of both spins. In the CSL mode the particle's state is influenced by the noise field. Over time, the particle will slowly settle into either the spin-up state or the spin-down state. When a measurement of the particle is made, one of these states is seen, depending on how the noise field has affected it (Pearle, 1989).

2. Discussion

The reasons why a definite interpretation has not been settled on by quantum physicists include the role the observer plays, the way that classical theory fails for both local and nonlocal quantum systems, as well as the apparent indeterministic and irreversible processes that are involved in quantum mechanics.

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The Copenhagen Interpretation

In this interpretation the observer plays a crucial role, which poses a problem. The system does not possess definite properties independently of measurements, it is the act of observation that brings the properties into existence (Bassi & Ghirardi, n.d.). This leads to the controversial notion that reality is somehow created upon observation. Another criticism for this interpretation would be that there is no clear mechanism for the collapse of the wavefunction, This indicates that the Copenhagen interpretation might be incomplete.

The Many-Worlds Interpretation:

One of the main paradoxes that has been struggled with by previous quantum mechanics interpretations is the multiple-observer's paradox.

The “multiple-observer paradox”

The presence of multiple observers during the process of measurement in a quantum system gives rise to this paradox: particularly, in the traditional interpretation of a quantum system, the act of measurement is considered a “special process” through which the wave function is caused to collapse. This process is intended to be carried out by a single observer without interaction with the quantum system; therefore, in a system with multiple observers, multiple measurements would result in multiple collapses of the same wave function, which would not be possible to explain within a traditional quantum mechanical interpretation.

A credible explanation of the multiple-observer paradox, in accordance with what is observed in reality, is suggested by Everett. The “many-worlds” interpretation, through the use of quantum entanglement in the description of a composite quantum system, is opposed to the classical description of a quantum system subject to the collapse of the wave function.

The observer-system relation as a composite quantum system

Two remarks are to be made in the implication that the observer-system relation can be regarded as a composite quantum system: there is entanglement between the observed system and the observer, and as a result, an independent state is held by neither of them. This result seems to be contradicted by real-life events: on the one hand, the final states are superpositions of many different states, each of which is associated with a definite observation outcome; on the other hand, there is only one outcome in real-life events.

The Transactional Interpretation:

Advantageously, in this interpretation, a clear visual model and intuitive explanations for quantum processes and phenomena is provided, and consistency with quantum mechanics and special relativity is maintained.

However, a problem is posed by the asymmetry of time. In classical mechanics, many fundamental equations are time-reversal symmetric, meaning they remain invariant under the transformation $t \to -t$. This indicates that these equations do not inherently distinguish between forward and backward directions of time. The acceptance of advanced waves (where solutions are reversed in time) is required for the transactional interpretation. (Cramer, 1986).

The Continuous Spontaneous Localization (CSL) Model:

Collapse models help explain why superposition isn't seen on a large scale. When a microscopic system (i.e. an electron) interacts with a macroscopic object, the wave function collapses. The CSL model provides a continuous explanation for this process, unlike the GRW model's sporadic events (Ghirardi et al., 1986).

One problem with the CSL model is the unlimited increase in energy induced by the collapse noise. This means that as the particle continuously interacts with the noise field, its energy keeps increasing without bound, which is not physically realistic (Bassi & Ghirardi, n.d.). Extensions to the CSL model, such as the dissipative CSL model, aim to address this by introducing mechanisms to conserve energy. An example of these mechanisms is shown in Equation (4.2).

$$d|\psi(t)\rangle = \left[-\frac{i}{\hbar}\hat{H}dt + \sqrt{\lambda}\sum_j \left(\hat{A}_j - \langle \hat{A}_j \rangle_t\right) dW_j(t) - \frac{\lambda}{2}\sum_j \left(\hat{A}_j - \langle \hat{A}_j \rangle_t\right)^2 dt - \frac{\gamma}{2\hbar}Ddt \right] |\psi(t)\rangle \tag{4.2}$$

Where γ is the dissipation rate parameter and D is the dissipative operator. D is chosen such that it counteracts the energy increase caused by the collapse noise. Typically, D is related to the momentum operators , since dissipation often involves momentum damping. The introduction of the dissipation term, $\frac{\gamma}{2\hbar}Ddt$, balances the energy input from the continuous interaction with the noise field, ensuring there is not unlimited energy increase (Diosi, 1989).

3. Conclusion

The Copenhagen Interpretation remains the most widely taught and accepted interpretation in the academic community. Its emphasis on the role of the observer and the probabilistic nature of quantum mechanics aligns well with experimental observations, despite ongoing debates and the existence of alternative interpretations. Different interpretations of quantum mechanics offer various explanations for the wave function collapse. The measurements performed in the same quantum system by different observers, would cause the same wave function to collapse multiple times, which is a contradictory phenomenon impossible to explain within a traditional description of a

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quantum system. Such contradiction is not present in Everett's theory as the wave function collapse phenomenon is completely disregarded in his interpretation, therefore, also the Copenhagen, the "hidden variables" and the "stochastic process" (i.e. "transactional theory") interpretations are invalidated by the many-worlds theory.

Everett's interpretation, however, lacks proof for fundamental concepts in quantum mechanics: for example, an adequate explanation for why specific outcomes are observed according to particular probabilities in experiments is struggled to be accounted for. However the CSL model allows for the understanding of how particles in quantum mechanics might collapse into a definite state in a continuous way, rather than suddenly and randomly. This aligns with how macroscopic objects don't exhibit quantum behaviours.

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"The Higgs Mechanism"

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Abstract

This research delves into the Higgs mechanism and its significant implications for cosmology, tracing the historical and theoretical development of the concept through the contributions of Peter Higgs and others. We examine the theoretical foundations that led to the prediction of the Higgs boson and its role in explaining how particles acquire mass. The study highlights the experimental confirmation of the Higgs boson by the ATLAS and CMS collaborations at the Large Hadron Collider, detailing the sophisticated detection techniques and the extensive data analysis involved. Furthermore, we explore the cosmological implications of the Higgs mechanism, particularly its impact on the early universe's evolution and the formation of large-scale structures. By integrating theoretical insights with experimental evidence, this research underscores the profound significance of the Higgs mechanism in contemporary physics and cosmology.

Introduction

The Higgs mechanism is a fundamental aspect of the Standard Model of particle physics, offering a crucial explanation for the origin of mass in elementary particles. The concept, first proposed by Peter Higgs and other physicists in the 1960s, has significantly advanced the understanding of the universe's basic structure. In 2012, the discovery of the Higgs boson by the LHC (Large Hadron Collider) experiments at CERN confirmed the theory, marking a historic milestone in science.

1. Properties of Higgs boson:

1. **Mass**: Approximately 125 giga-electron volts(GeV/c^2). It was confirmed by experiments at the Large Hadron Collider in 2012. $m^2H=2\lambda v^2$. Substituting $v\approx 246$GeV, the mass mH is approximately 125 GeV, as experimentally measured.
2. **Spin**: It has spin 0. The Higgs boson is a scalar particle, meaning it has no intrinsic angular angular momentum.
3. **Charge**: It has 0 electric charge. It is electrically neutral.
4. **Parity**: Its parity is +1. The Higgs boson is a scalar field excitation, which contributes to its positive parity.

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5. **Decay Modes**: It decays into various particles, including photons, W and Z bosons, and fermions like quarks and leptons.

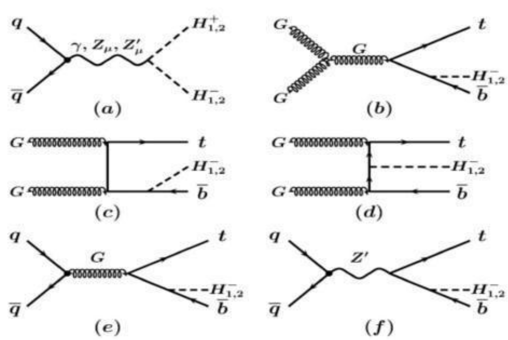

Figure 1

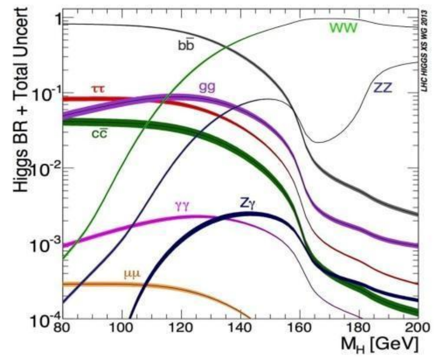

Figure 2

Figure 1: The Standard Model Higgs boson decay branching ratios and total width.

Figure 2: Charged Higgs boson production in pp collisions for (a) pair production in quark- antiquark annihilation, (b)-(d) associated single production in gluon-gluon collision and (e) – (f)

2. The Higgs mechanism:

The Higgs mechanism is one of the most important discoveries of modern physics, since it answers a very fundamental question: why do particles have mass?
Before answering this question, let's see why we need a Higgs mechanism or field in the first place. The problem arises from the electroweak interaction: according to a previous model, particles described by this theory should not have any mass, in order to respect some important symmetries, the gauge symmetries. These are transformations of some mathematical parameters that leave the physical description of a phenomenon unchanged [13]. Nevertheless, we observe from experimental data that the W and Z bosons (vectors of the weak interaction) have mass! How can we solve this paradox?
Let's start from a very important equation in quantum mechanics, the Klein-Gordon equation [12]:

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$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2c^2}{\hbar^2}\right)\phi = 0 \qquad [1]$$

This is equation is related to Einstein's relativistic energy-momentum relation $\left(E^2 = p^2c^2 + m^2c^4\right)$: it contains the most important information about a quantum particle or field, such as its energy, momentum and mass. ϕ is a quantum wave function, but since we are in the domain of quantum field theory, here it is a general quantum field. In this case, m is the mass of the excitation of the field (i.e. its particles).

Now, let's consider two interacting fields ϕ_1, whose particles are massless like the electroweak theory tells us, and ϕ_2. The Klein-Gordon equation for the first one is

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + k_{1,2}\phi_2^{\ 2}\right)\phi_1 = 0 \qquad [2]$$

Notice that since its particles have no mass the mass-related term has disappeared and the new term $k_{1,2}\phi_2^{\ 2}$ accounts for the interaction between $\phi_1$and ϕ_2.

As we can see from equation [2], although there is not the mass-related term anymore, we still have a positive addend term that plays the same "mathematical role" as the previous one. We just need to make sure that $k_{1,2}\phi_2^{\ 2}$ is a non-zero term. To do that, let's consider the Klein-Gordon equation for ϕ_2:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m_2^{\ 2}c^2}{\hbar^2}\right)\phi_2 = 0 \qquad [3]$$

The problem is that the only solution here is $\phi_2 = 0$. So, this field must have some particular characteristics. Let's consider a field interacting with itself:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m_2^{\ 2}c^2}{\hbar^2} + k_{2,2}\phi_2^{\ 2}\right)\phi_2 = 0 \qquad [4]$$

As for the equation [2], the $k_{2,2}\phi_2^{\ 2}$ term accounts for the interaction with itself. Now we have to find the solution to this equation. One of them is again $\phi_2 = 0$. In order for the solution to be stationary, $\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\phi_2$must be equal to 0. Therefore the other solutions are found calculating $\frac{m_2^{\ 2}c^2}{\hbar^2} + k_{2,2}\phi_2^{\ 2} = 0$. This is a quadratic equation with solution $\phi_2 = \pm\sqrt{-\frac{m_2^{\ 2}c^2}{\hbar^2 k_{2,2}}}$. Now we have three solutions for ϕ_2. In order to make sure which one we can accept we have to understand how the potential energy U of the

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system varies as a function of ϕ_2. It turns out that the null solution corresponds to a maximum value of the potential energy, i.e. an unstable condition. Hence, we can discard the $\phi_2 = 0$ option [21].

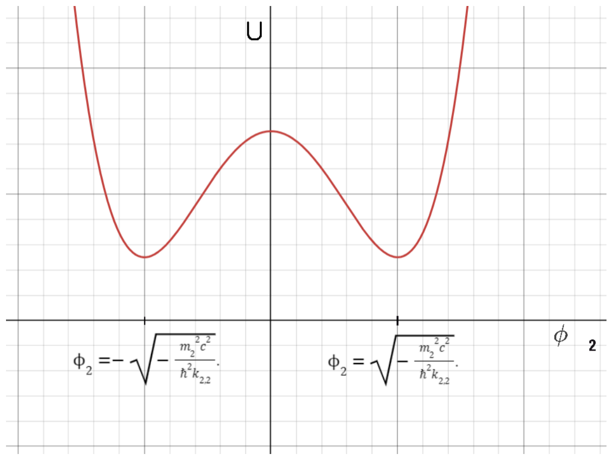

Figure 3. Graph of potential energy U against ϕ_2 values

Now we are sure that the $k_{1,2}\phi_2^{\ 2}$ term in equation [2] is non-zero. As a result, after the interaction with ϕ_2, ϕ_1 does get mass! ϕ_2 is the Higgs field. Furthermore, the assumption that it is self-interacting is actually true: we have discovered that the Higgs boson has mass and that means that it interacts with its own field.

3. Higgs boson and its implications

The Higgs boson is a cornerstone of the Standard Model of particle physics which describes the fundamental particles and forces in the universe. It helps in understanding the conditions of the universe and the evolution of the cosmos. Some properties of the Higgs boson could have implications for cosmological models relating to dark matter and dark energy. It opens new avenues for research which might eventually lead to new discoveries and astonishing theoretical insights.

4. History of the research:

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The idea of spontaneous symmetry breaking, pivotal for the development of the Higgs mechanism, was first introduced by Yoichiro Nambu in 1960, but it was related to superconductivity [1].

Every quantum field has a potential energy, which reaches its minimum in its "vacuum state". This term is not related to emptiness, because even in the case of an absence of physical particles, "virtual" particles must be considered.

A graph representing the force of a field on the x axis and the potential in the y axis, until the end of the Electroweak Epoch (10 to the minus twelfth power seconds after the Big Bang) had a parabolic shape. As the temperature of the universe descended, the electroweak phase transition was provoked, changing the graph shape to another symmetry but with two minima or "vacuum states". The impossibility of being at the two vacuum states at the same time caused the symmetry to break: a spontaneous symmetry breaking. However, the potential of the Higgs field has another dimension and is called a "mexican hat potential".

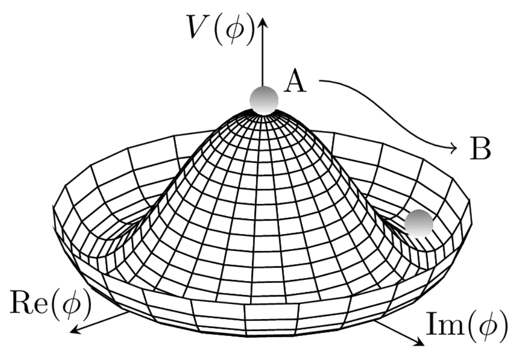

Figure 4. Diagram of the potential of the Higgs field.

The phase of the field (its angular position in the brim of the "mexican hat") is not determined and a change in the phase would not cost energy. From here emerges a global symmetry under rotational fluctuations. A theorem proved in 1961 by Jeffrey Goldstone, Abdus Salam and Steven Weinberg, states that spontaneous breaking of a global symmetry leads to the appearance of one or more
Nambu-Goldstone bosons, presumably massless and spin-zero [1].

In the same year, Julian Schwinger observed those particles were not massless. In 1962, Philip Warren Anderson demonstrated the emergence of massive particles but his theory worked under non-relativistic terms. In 1964, three groups worked on a relativistic theory. However, only Peter Higgs proposed the existence of a massive particle associated with the scalar field [1].

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Peter Higgs (1929-2024) received a bachelor's degree, master's degree and doctorate in physics from King's College London. Although his earliest work belongs to molecular physics, in 1956 he began his work on quantum field theory. He received numerous honours for his work, culminating in a Nobel Prize in 2013 together with François Englert.

The experimental search of the Higgs boson started in the 80s, limited by the availability of accelerators. Research conducted at DESY (Deutches Elektronen Synchrotron) in 1984 observing the upsilon decay ($Y \rightarrow H + \gamma$) and other ones in that decade concluded that the Higgs mass was larger than 9 $\frac{GeV}{c^2}$ [2].

At CERN, research using the Large Electron-Positron Collider conducted between 1989 and 2000, observed a series of events resembling a Higgs boson with a mass of 115 $\frac{GeV}{c^2}$ in the ALEPH experiment. However, the results were inconclusive due to the lack of evidential support from other experiments at LEP [3].

5. LHC, discovery of the higgs boson and the role of ATLAS and CMS:

The Higgs boson was discovered in 2012 nearly 50 years after it was first theorised. The long search for this particle is due to its significant mass, over 120 times that of a proton, and its extremely short lifespan, lasting only 10^-22 seconds. This means the Higgs boson cannot be found naturally and must be produced in a laboratory, specifically in a particle collider like the Large Hadron Collider (LHC), the world's largest and most powerful particle accelerator, located near Geneva, which began operating in 2010.

The ATLAS (A Toroidal LHC Apparatus) experiment (figure 5)[6], is one of the principal pillars of the LHC. ATLAS is a multipurpose particle detector with a forward–backward symmetric, cylindrical geometry and a near 4π coverage in solid angle. The detector records digitised signals produced by the products of LHC's proton bunch collisions, hereafter termed collision 'events'.

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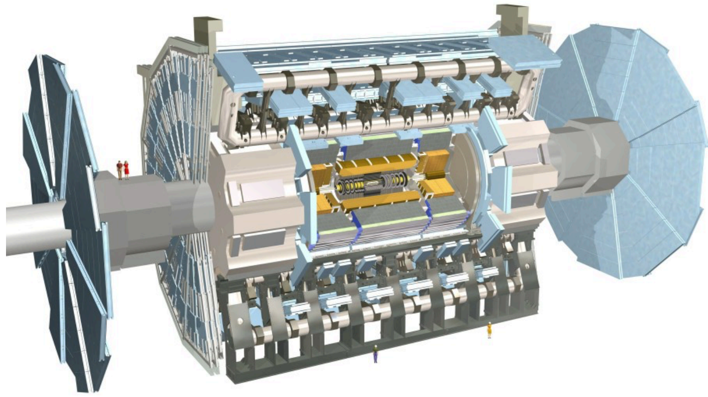

Figure 5. Section of the Atlas Experiment - Vincent Hedberg, CERN

The research entails a significant challenge in discriminating between Higgs boson decay products and those originating from a multitude of concurrent collision events, owing to its rarity of occurrence [5]. By the time of the Higgs boson discovery announcement, scientists analysed data from around 300 trillion (3×10^{14}) proton-proton collisions [4]. Out of these collisions, the number of Higgs boson events observed was 500, highlighting the challenge of detecting such a rare particle.
Scientists analyse many collisions and measure a quantity called invariant mass from the detected particles. For Higgs decay products, this mass matches the Higgs mass consistently. By analysing a large number of collisions, we can identify a slight excess at a specific invariant mass, indicating the Higgs boson's presence.

On 15 June 2014 ATLAS physicists produced their first measurement of the mass of the Higgs boson [10]. Their paper studied all available LHC collision data at that time, looking at the Higgs boson in its decays to two photons and into four leptons [7].

The decay of the Higgs boson (figure 6) [4][9] encompasses several distinct channels, each revealing crucial insights into its interactions and properties. In the γγ decay channel, the Higgs boson transforms into two photons, offering a pristine signature characterised by precise measurements of photon energies and directions. In the ZZ decay mode, the Higgs boson decays into two Z bosons, which subsequently decay into leptons or quarks. Detection of four-lepton or four-quark final states facilitates the identification of this decay pathway. The WW decay channel involves the decay of the Higgs boson into two W bosons, each of which further decays into leptons or quarks and neutrinos. This channel is distinguished by the detection of two leptons accompanied by missing energy from neutrinos. The bb decay mode is prominent, with the Higgs boson

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frequently decaying into a bottom quark and its antiparticle, providing valuable insights into its interaction with fermions. Furthermore, the Y decay channel, where the Higgs boson decays into a pair of Y, also contributes to our understanding of its interactions with heavy quarks.

Decay mode	Targeted production processes	$\mathcal{L}$ [fb^{-1}]	Ref.	Fits deployed in
$H \to \gamma\gamma$	ggF, VBF, $WH, ZH, t\bar{t}H, tH$	139	31	All
$H \to ZZ$	ggF, VBF, $WH + ZH, t\bar{t}H + tH$	139	28	All
	$t\bar{t}H + tH$ (multilepton)	36.1	39	All but fit of kinematics
$H \to WW$	ggF, VBF	139	29	All
	WH, ZH	36.1	30	All but fit of kinematics
	$t\bar{t}H + tH$ (multilepton)	36.1	39	All but fit of kinematics
$H \to Z\gamma$	inclusive	139	32	All but fit of kinematics
$H \to b\bar{b}$	WH, ZH	139	33,34	All
	VBF	126	35	All
	$t\bar{t}H + tH$	139	36	All
	inclusive	139	37	Only for fit of kinematics
$H \to \tau\tau$	ggF, VBF, $WH + ZH, t\bar{t}H + tH$	139	38	All
	$t\bar{t}H + tH$ (multilepton)	36.1	39	All but fit of kinematics
$H \to \mu\mu$	ggF + $t\bar{t}H + tH$, VBF + $WH + ZH$	139	40	All but fit of kinematics
$H \to c\bar{c}$	$WH + ZH$	139	41	Only for free-floating κ_c
$H \to$ invisible	VBF	139	42	κ models with $B_{\text{u.}}$ & $B_{\text{inv.}}$
	ZH	139	43	κ models with $B_{\text{u.}}$ & $B_{\text{inv.}}$

Figure 6. Listed are the measured decay modes, targeted production processes and integrated luminosity (L) used for each input analysis of the combination.

6. Higgs field in cosmology:

Cosmic inflation was proposed in 1981 by Alan Guth. Inflation posits an exponential expansion driven by a scalar field with a dominating potential energy in the early universe. The model of Higgs inflation associates the inflation with the Standard Model Higgs boson, thereby bridging cosmology with particle physics. The Higgs boson, which provides mass to elementary particles through the Higgs mechanism, is linked to the Higgs field, a scalar field responsible for this mass generation[15].

The conditions that need to be excited for the Higgs field to cause inflation are:

The potential Higgs field:

The potential for the Higgs field in the Standard Model: $V(h) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2$

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λ is the self-coupling constant, v expected vacuum, h is the Higgs field. But the Higgs Must have a shape that allows slow-roll conditions to happen, this means the potential needs to be flat rather than quickly falling downslope.

Coupling to Gravity:

For the Higgs field to drive the expansion of the early universe requires interaction with gravity, This coupling changes the way the field interacts with the fabric of spacetime, allowing it to drive inflation effectively. For observing the inflationary consequences of SM Higgs boson minimally coupled to gravity through the spacetime metric alone with no interaction between the space curvature of spacetime, for inflation to occur the Higgs field must roll slowly down its potential, Hence the potential must be flat enough to satisfy slow- roll conditions, in minimal coupling the potential tends to be steep to sustain a long period of inflation[16], yet the universe doesn't get enough time to expand as much as it needs to during inflation. For a successful assumption between the Higgs inflation and the SM including the Higgs field must be Non-minimal coupled to gravity. This means the field has an interaction with the curvature of space and time (Ricci scalar R). This idea proposed by Bezrukov & Shaposhnikov (2008).Non-minimal coupling modifies the effective potential of the Higgs field making it flatter[14], The flattened effective potential allows the Higgs field to roll slowly, satisfying the slow-roll conditions of a long period of inflation.

7. Discussion

The Higgs mechanism and studies of the kinematics of the production of the Higgs boson have proved to be in agreement with the predictions of the Standard Model according to the experiments performed in ATLAS from 2015 to 2018 [19]. Nonetheless, measurements of some of its properties, specifically the coupling of the Higgs boson to itself, still need to be done [19].

The Higgs mechanism has successfully explained the generation of mass in elementary particles. However, this understanding remains incomplete until it includes the explanation of the appearance of mass in dark matter. The idea of the spontaneous

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symmetry breaking, explored in this research, could also be crucial to attain it, as suggested in recent research [18], which also includes potential phenomenological studies of dark matter. Dark Higgs field constitutes a new possibility in the Higgs mechanism research.

Recent research [20] proposes a reinterpretation of the Higgs mechanism, connecting it with quantum information theory and creating an interesting path of new ideas. The research creates a mathematical formalism that relates the Higgs potential, explored in depth throughout this text, with quantum entanglement entropy.

8. Conclusion

Our research has thoroughly investigated the intriguing Higgs mechanism and its profound impact on our understanding of the fundamental constituents of matter. We have explored the Higgs boson's properties, its underlying mechanism, and the historical journey leading to its discovery. Starting with the theoretical framework, we delved into the properties of the Higgs boson and the mathematical principles that underpin its mechanism, offering a comprehensive understanding of mass acquisition and the Higgs boson's interaction with its own field.

The theoretical explanations were complemented with a historical review of the ideas that laid the groundwork for the Higgs mechanism. This historical context culminated in the experimental detection of the Higgs boson at CERN’s LHC in 2012. By examining the experimental methods and the characteristics of the ATLAS detector, we highlighted the technical prowess required to identify this elusive particle, focusing on the decay channels of the Higgs boson and the sophisticated detection techniques used in this groundbreaking discovery.

Additionally, we have delved into the significant implications of the Higgs boson in cosmology. By linking the Higgs mechanism with inflation theory, we demonstrated how it enables the development of new cosmological ideas that explain the expansion of the universe. This connection underscores the Higgs boson's role not only in particle physics but also in enhancing our understanding of the universe's evolution and structure.

While substantial progress has been made, numerous questions remain unanswered. Ongoing investigations in Higgs physics continue to drive us toward new insights and potential breakthroughs, promising to further unravel the mysteries of our universe. The

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continuous study of the Higgs boson not only enhances our grasp of the fundamental nature of matter but also opens new avenues in cosmology, fostering a deeper comprehension of the universe's dynamics.

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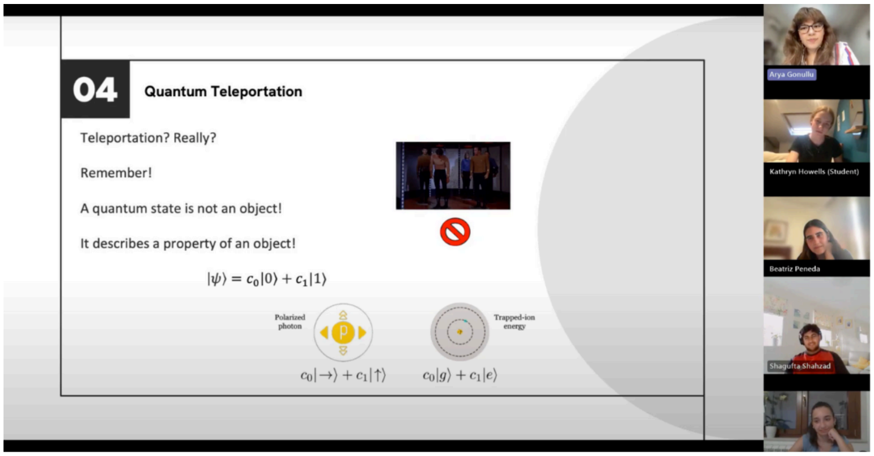
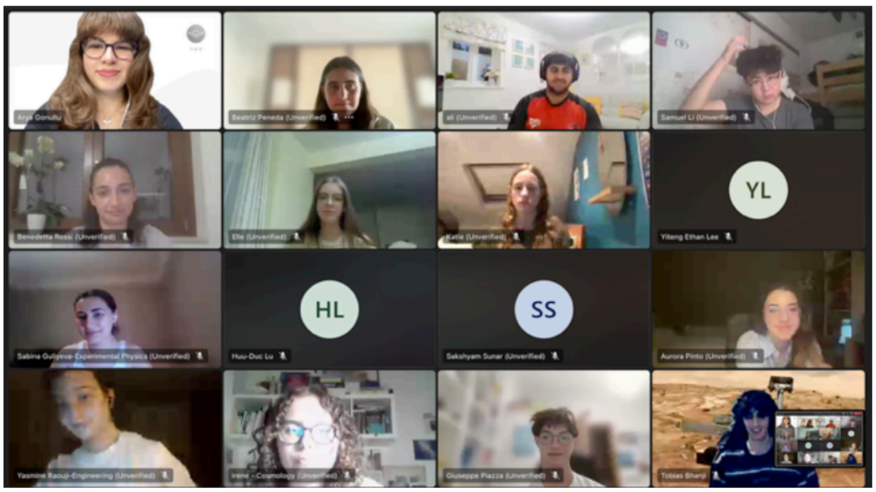
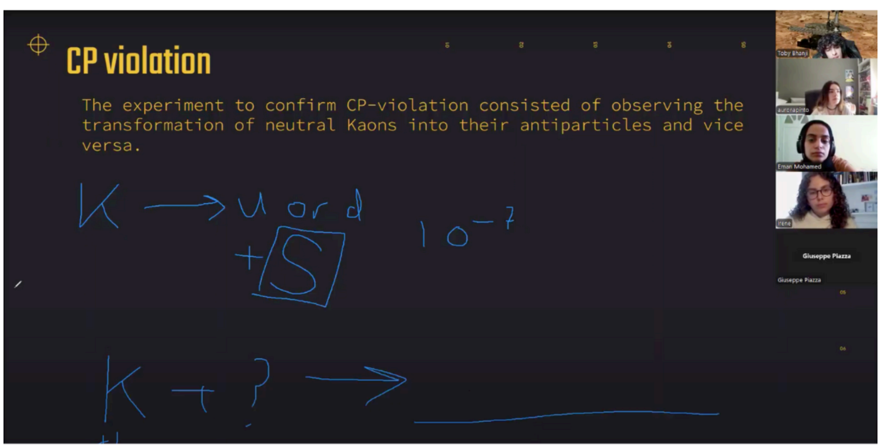


Quantum Mechanics Lecture 5



Final Day Presentations



Cosmology Lecture 2

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“The Search for Magnetic Monopoles”

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Abstract

Magnets always share a common characteristic: they have two poles. Even though their division is considered, the result remains a smaller dipole. Nevertheless, physicists have been searching for a magnetic monopole. It is a hypothetical particle predicted by several theories. The magnetic monopole supposed a symmetry between electric and magnetic fields. The charge would be quantized in discrete units if this concept is proved. Experiments for their detection take place at CERN, specifically in the Large Hadron Collider (LHC).

01. Introduction

Magnetic monopoles were first introduced by Paul Dirac in 1931.The existence of a magnetically charged particle would add symmetry to Maxwell's equations and explain why electric charge is quantized in nature(Acharya et al., 2019,2) opening new directions to research in electromagnetism. Despite any theoretical approach, numerous experiments have been conducted to search for this particle; however, many have yet to succeed. Currently, LHC is focusing some of its experiments, such as ALICE, a heavy-ion collider, ATLAS, a general-purpose detector, and MoEDAL, designed to search for monopoles and other exotic particles, on detecting monopoles. The LHC is primarily using three methods to detect monopoles. The first method involves producing pairs of magnetic monopoles in particle interactions, either from a single photon—a technique known as the Drell-Yan mechanism—or from the fusion of two photons. The second method, known as the Schwinger mechanism, is based on producing pairs of magnetic monopoles from the vacuum in the intense magnetic fields created when heavy ions nearly collide. And the last one is the photon-fusion mechanism. ATLAS experiments results have benefited compared to previous searches, due to larger datasets.

This paper reviews the current state of research in monopole detection, focusing on its current detection, the fundamental theory of magnetic monopoles, and previous and ongoing research in the field.

02. Mathematical and physics implications.

This paragraph serves the purpose of explaining the mathematical and physics concepts behind the hypothesis of monopoles delving deeper also on the theoretical tradition that was present before this supposition.

It is possible to describe the magnetic force in a moving particle by using the vectorial product of the velocity vector, the magnetic field, the electric field, and the charge module.
This force is also called Lorentz force and it's written in the following way:

$$F = q(E + v \times B)$$

(Equation 1)

However consider the equation only in a magnetic field this time, which leads it to be,

$$F = qv \times B$$

(Equation 2)

Moreover, the magnetic force is a non-conservative force, with this information in hand we can state that since the force is always perpendicular to the velocity vector, which is also the direction of the motion; it means that the Work that the force does to move the charge is $L = F \cdot \Delta x$ and, since Δx is parallel to v and F is perpendicular to both of them, work is the scalar product of two vectors that are perpendicular to each other, in conclusion, the equation is null.
This means that if no work is put upon the charge, the module of the velocity is constant.
In case the charge has a circular trajectory, then it could be considered a circular motion, and the magnetic force is a centripetal force.

$$qvB = m(v^2/r)$$

(Equation 3)

$$1/qB = r/mv$$

(Equation 3.1)

$$r = mv/qB$$

(Equation 3.2)

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This equation is necessary to experimental physicists as it helps in experiments and applications such as mass spectrometers (to separate various isotopes due to the difference in their mass), magnetic flowmeters (mainly used in medicine), and also in speed selectors.

An important but apparent aspect of magnets is that t*hey* always have two poles, a north and a south.
We can think about how when a magnet is divided, thanks to the concept of magnetization, due to the positions that the electrons take inside the magnets, there will always be two newly generated poles. To this concept, every single electron inside of a metal becomes a small magnet that reacts and aligns with the field.

But, only certain materials exhibit strong magnetic effects, these metals are called ferromagnetic.
In this category, it is possible to find chemical elements such as iron, cobalt, nickel, and gadolinium.
Instead, diamagnetism is the effect of the production, on behalf of a material, of a direct magnetic field but in the opposite direction of the external *B* field. Meanwhile, paramagnetism stands out since its elements do not have their own magnetic field; unless an Electromagnetic field is applied externally.

The mathematical concept that sets these three phenomena apart, is the *relative magnetic permeability.*

$$\mu r = \mu / \mu 0$$

(Equation 4)

- μ $is\ the\ permeability\ of\ a\ substance.$
- $\mu 0$ $is\ the\ permeability\ in\ a\ vacuum.$

These pieces of information were considered a statement until Dirac formulated the concept of magnetic monopoles.
The magnetic monopole was hypothesized to explain a specific symmetry between *E* and *B* fields, therefore the monopole should be the equivalent of the electric point charge.

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Moreover, if the concept of a monopole is proved this means that electric charge is necessarily quantized in discrete units.

The mathematical implication of said statement is the quantization equation:

$$e \cdot g = (n\hbar)/2$$

(Equation 5)

Where: *e* is the electric charge, *g* is the magnetic charge, $\hbar$ is the reduced Planck's constant ($\hbar$= *h* /(2π)) (Equation 6), n is an integer number (a number that belongs to the mathematical set N.
This condition implies that the product of the eclectic and magnetic charge must be quantized in units of $\hbar/2$.

The mathematical implications of these equations are mainly three.
The theory of charge quantization, which is explained formerly, the gauge symmetry, highlights the deep connection between these symmetries and the topological properties of space and the concept of duality and how it furthers the hypothesis of the GUTs (grand unified theories).
In the next paragraph, it will be explained how such a physical concept is produced in world-leading experiments such as MoEDAL in LHC.

03. How magnetic monopoles are generated at LHC

Although magnetic monopoles have never been detected at the MoEDAL experiment at CERN, the experiment aims to detect monopoles generated by three different mechanisms: the Drell-Yan process, the photon fusion mechanism, and the Schwinger mechanism. These are all processes that occur during high-energy Pb-Pb collisions.

In all three processes of generating magnetic monopoles, the monopoles would appear due to a vibration in the magnetic monopole quantum field where energy is converted to matter. As mass is directly proportional to energy via Einstein's $E = mc^2$ equation, and also considering the fact that a magnetic monopole is predicted to be around 4700x the mass of a proton, in order to generate a particle of such high mass it would require a huge amount of energy. Thus, high-energy heavy ion collisions are studied.

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Role of LHC in the acceleration of Pb ions

In order for the Pb-Pb collision to have sufficient energy to trigger the mechanisms mentioned above they must be accelerated to 99.9999991% of the speed of light. LHC is the final and most integral part of this process. Before being injected into LHC the Pb ions are accelerated by a series of different accelerators. To a final energy of 177 Gev per nucleon which also strips the ions of all their electrons. In LHC, the particles are accelerated by radiofrequency (RF) cavities. LHC consists of 16 RF cavities. The RF cavities are driven by electron beams that oscillate 400 million times per second. A conductive pipe called a waveguide directs energy to the cavity which is designed to allow intensity electromagnetic waves to build reaching a maximum of 16MV per beam. The RF cavities are housed in cryomodules which keep them at a sufficiently cool temperature to maintain superconductivity.

The lead ions colliding in LHC are initially Pb 208 atoms. They begin as a 2cm 500mg strip of pure lead which is heated to 500 degrees Celsius to vaporize a small number of atoms, the first few electrons of which are ionized using an electric current. The electromagnetic waves produced by the RF cavities increase the energy of the incoming particles by more than 14 times. The particles pass through the RF cavities more than 100 million times over the course of about 20 minutes before reaching maximum speed.

During LHC heavy ion collisions, it is theorised that magnetic monopoles can be generated via three different mechanisms: The Drell-Yan process, the photon fusion mechanism and the Schwinger mechanism.

The Drell-Yan process occurs during these high-energy collisions in high-energy hadron-hadron scattering which comes from the interactions between the wavefunctions of the particles. The collisions also trigger the annihilation of a quark from one hadron and an antiquark from another hadron. This creates a virtual photon.

A virtual photon is a fluctuation in the quantum photon field. The vibration required to produce a real photon is similar to that of a virtual photon however it is more regular and more permanent.

In the Drell-Yan process, a magnetic monopole particle-antiparticle pair can be created by a virtual photon decaying into these particles. This has not been observed.

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The photon fusion mechanism happens under the same conditions as stated above in reference to the Drell-Yan process. However in this case the virtual photons interact, involving interactions between vibrations in the quantum field) and this produces a magnetic monopole particle and antiparticle pair.

The Schwinger mechanism occurs during near-miss heavy ion collisions. When heavy ions such as lead engage in near-miss collisions the protons and neutrons (with positive and neutral charges respectively) that do not collide would be set aswirl generating the strongest known magnetic fields in the current universe as they're traveling at speeds very close to the speed of light. Pairs of magnetic monopoles could be produced from the vacuum in the magnetic field.

04. Detection of Magnetic monopoles

Despite the theoretical appeal of magnetic monopoles, scientists have developed several methods to try to detect them. The quest to detect magnetic monopoles through electromagnetic induction has been a fascinating journey in experimental physics. Current research in monopole detection encompasses a variety of techniques, each with its own strengths and limitations. These methods span different energy scales and leverage diverse physical phenomena, reflecting the multifaceted nature of the monopole search. Some of the prominent detection techniques under investigation include:

4.1 SQUIDs

Scientists first discovered the idea of directly observing magnetic charges using electromagnetic induction. The principle was elegantly simple: a magnetic monopole passing through a closed conducting ring would induce a persistent change in current, as the system attempted to maintain its original magnetic flux. This simply would not happen in the case of a magnetic dipole or higher order magnetic pole, for which the net induced current is zero, and hence the effect can be used as an evident test for the presence of magnetic monopoles. Initial experiments utilized room-temperature conducting coils, laying the groundwork for future developments.

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The introduction of superconducting technology revolutionized magnetic monopole detection. Superconducting rings provided a key benefit: the capacity to maintain induced current changes indefinitely without the Joule heating inherent in conventional conductors. This innovation enabled more sensitive, longer-duration experiments. However, early superconducting detectors were limited by the inadequate sensitivity of available current-measuring electronics. This constraint required multiple passes of potential monopole-containing samples to produce detectable signals, restricting initial investigations mainly to bulk matter searches. Despite these challenges, superconducting technology laid the foundation for more advanced monopole detection methods.

The true revolution in monopole detection came with the development of the Superconducting Quantum Interferometer Device (SQUID) and ultra-low magnetic field shields. These technologies dramatically enhanced the sensitivity and precision of monopole detection methods. SQUIDs, capable of detecting incredibly minute magnetic fields, opened the door to dynamic monopole detection. This advancement allowed researchers to move beyond static bulk matter searches to more versatile and sensitive detection methods.

In 1982, physicist Blas Cabrera made a potentially groundbreaking discovery where Cabrera's custom-built detector registered a signal consistent with the existence of a magnetic monopole, which was described as a hypothetical particle that had never been observed before. Cabrera had spent three years developing and fine-tuning his experimental apparatus to detect these elusive particles. The detector was designed to be extremely sensitive, capable of registering the minute magnetic field that a monopole would produce as it passed through the device. Despite the initial enthusiasm this event has caused, the Cabrera event remains an isolated incident, where since that discovery, neither Cabrera nor any other researcher has been able to replicate the result or detect another magnetic monopole. This lack of corroboration has led to ongoing debate about the nature of the original signal.

4.2 Material-Specific Searches

The search for magnetic monopoles in matter and within our Solar System represents a fascinating chapter in modern physics, blending theoretical predictions with innovative experimental approaches. This quest has led scientists to explore a variety of potential sources and trapping mechanisms for monopoles.

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At the core of this search is the possibility that monopoles could be present in ordinary matter. This presence could arise from two primary mechanisms: accretion during the formation of matter or the stopping of monopoles after they lose kinetic energy. The behavior of these particles in matter is theorized to be complex, with potential binding to ferromagnetic or paramagnetic materials through image charges, and even possible interactions with atomic nuclei.

Spin ice is a class of magnetic materials that has gained significant attention in the field of condensed matter physics, particularly in relation to the study of magnetic monopoles. These materials, typically rare-earth titanates such as Dy2Ti2O7 or Ho2Ti2O7, exhibit fascinating magnetic properties at low temperatures due to their unique crystal structure and magnetic interactions. The crystal structure of spin ice materials consists of corner-sharing tetrahedra, where magnetic rare-earth ions (such as Dy3+ or Ho3+) occupy the vertices. The magnetic moments of these ions are constrained by crystal field effects to point either directly toward or away from the center of each tetrahedron, resembling the arrangement of hydrogen atoms in water ice (hence the name "spin ice"). This geometric arrangement leads to magnetic frustration, a phenomenon where the system cannot simultaneously satisfy all pairwise magnetic interactions. This frustration results in a highly degenerate ground state with residual entropy, analogous to the residual entropy in water ice discovered by Linus Pauling.

The connection between spin ice and magnetic monopoles arises from the concept of emergent excitations. Castelnovo, Moessner, and Sondhi proposed that the elementary excitations in spin ice could be described as emergent magnetic monopoles. When a single spin in the spin ice structure is flipped, it creates a pair of defects that can be interpreted as a north and south magnetic monopole pair. These monopoles are not fundamental particles like those sought in high-energy physics experiments (such as Cabrera's), but rather quasiparticles – collective excitations that behave effectively as monopoles within the material.

In planetary bodies, the distribution of monopoles is expected to follow gravitational forces, leading to a concentration near the core. This expectation has prompted searches in various terrestrial contexts. One intriguing approach involves examining iron from the Earth's surface, though the uniform distribution of monopoles throughout the planet's interior complicates this method. A more novel strategy targets iron refinery

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operations, where vast quantities of ore are heated above the Curie point annually, potentially allowing trapped monopoles to fall out under gravity.

The detection of the presence of monopoles extends beyond Earth to other celestial bodies in our Solar System. Meteors, with their smaller gravitational fields, have been proposed as potential harbors for monopoles. However, the high momentum of monopoles presents a challenge, as any meteor-trapped monopole striking Earth would likely pass through unimpeded.

Perhaps the most audacious proposal in this field is the concept of solar monopoles. This hypothesis, partly motivated by unexpectedly high flux events like the Cabrera observation, suggests that the Sun could contain a vast number of monopoles and regularly emit them. According to this model, solar flares might expel monopoles at velocities comparable to Earth's orbital speed, creating a cloud of these particles in Earth's orbit. Intriguingly, this solar monopole model uniquely accommodates the 11-year sunspot cycle in its flux predictions, adding an extra layer of intrigue to the hypothesis.

However, the solar monopole theory faces significant challenges. Calculations indicate that the concentrating effect for Grand Unified Theory (GUT) monopoles in the Sun is likely insufficient to achieve the proposed solar population. This discrepancy highlights the ongoing tension between theoretical predictions and observational constraints in the field. The diverse approaches to monopole detection in matter and our Solar System underscore the creativity and persistence of the scientific community.

05. Previous Experiments and Techniques at CERN

These particles are extremely difficult to detect and extensive experimentation along with state-of-the-art techniques are used at the European Organization for Nuclear Research or CERN. Among these, the experiments include ATLAS and MoEDAL detectors and heavy ion collisions. I will make these notes in this discussion regarding the earlier experiments and techniques at CERN: the contribution of ATLAS and MoEDAL, heavy ion collisions, and the possibilities of monopole detection at HL-LHC.

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5.1) ATLAS Detector:

- Picking on the LHC big infrastructural apparatus, ATLAS (A Toroidal LHC Apparatus) is one of the largest and most flexible detectors. Its main objective is to scan a spectrum of physics subjects, from the detection of magnetic monopoles.
- In ATLAS numerous specific searches for monopoles have been performed with the utilization of vast samples of data obtained with high-energy proton-proton interactions. This was accompanied by tracking systems and have greatly improved the sensitivity to rare and exotic particles.
- However, the searches for monopoles have not been successful and have not provided the aspects of monopoles up to date but the searches have set certain cross-sections and masses of monopoles which is beneficial for the field of science.

5.2) MoEDAL Detector:

- MoEDAL – Monopole and Exotics Detector at the LHC are going to search for magnetic monopoles and other highly ionizing particles.
- Several methods to identify monopoles are used in MoEDAL; they include the use of plastic nuclear track detectors and trapping volumes.
- In the absence of any monopole discoveries so far, MoEDAL has expanded the scope of the detected capacities and given fairly tight constraints on various theories.

5.3) Heavy Ion Collisions:

- The heavy ion program of the LHC especially by the ALICE experiments explores the appearance of monopoles in gargantuan magnetic fields created during heavy ion collisions.
- As these experiments did not find any monopoles, the work is perfecting the detection methods and giving important data for subsequent studies.

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06. Future Plans and Prospects at CERN

However, as CERN moves forward in their quest for magnetic monopoles the future looks even more promising. The new High-Luminosity LHC (HL-LHC) is slated to boost collision frequencies, improving the possibility of seeing strange phenomena such as monopole creation. The subsequent improvement of key detectors like MoEDAL and designing new techniques will improve the search to provide extraordinarily high sensitivity and a precise outcome.

6.1) High-Luminosity LHC (HL-LHC):

Expected to start operation in the mid-2020s, the HL-LHC will provide a much higher interaction rate at the LHC thereby providing better chances for the observation of the otherwise rare phenomena like the monopole production.

The HL-LHC should produce ten times more data than LHC, and thus improve the monopole search sensitivity by one order of magnitude.

Better tracking and resolution of new upgraded versions of the ATLAS and MoEDAL detectors will be the key to these endeavors.

6.2) Upgraded MoEDAL Detector:

Several improvements to MoEDAL will be made to improve the detector sensitivity; New nuclear track detectors, New Timepix pixel detectors, and New data acquisition systems.

These improvements can be useful for experiments and for enhancing identification and characterization of the monopole events in the MoEDAL detector.

Advanced Detection Techniques:

Methods for the enhanced identification of monopoles are being devised to facilitate detection at CERN at the present moment. Some of them are high-precision timing detectors, and track structure sensors which are used to filter and sort monopole signals from other interfering sources.

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Moreover, with the advancements in the algorithm and machine learning, there will be advancements in the identification methods of the event of monopole.

Conclusions

This paper explored the theoretical foundations and experimental approaches for magnetic monopoles, tracing the journey from early theoretical proposals to modern particle accelerator technology. The quest to detect magnetic monopoles began in the 1960s when scientists suggested observing them experimentally. The ATLAS, MoEDAL, and ALICE experiments, each employing different methods to transform energy into matter, represent significant efforts in this pursuit. While these experiments have not yet detected magnetic monopoles, they have advanced both experimental and theoretical boundaries in electromagnetism.

These pioneering experiments have paved the way for future studies at the Large Hadron Collider (LHC), with prospects of higher collision rates, increased data collection, and improved detection techniques. Such advancements will enhance the chances of discovering monopole particles. Future investigations could extend to exploring alternative experimental setups, integrating cutting-edge technology, and collaborating across different research fields to broaden the search for magnetic monopoles.

Overall, the experiments have been successful in contributing substantial knowledge and methodological innovations to the field of electromagnetism, despite yet to achieve direct detection of magnetic monopoles.

Author Contributions:

- Abstract, introduction - Susana C.
- Mathematical and physics implications - Yasmine R.
- Generation - Eleanor M.
- Detection - Judy A.
- Previous Experiments and Techniques - Sabina G.
- Future Plans and Prospects - Sabina G.
- Conclusions - Susana C.

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"Explorations in Graph Theory, PageRank & AI"

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1. PageRank Algorithm

Introduction

This section of the research paper will detail the history of the PageRank algorithm specifically with Google and briefly cover the development process.

History of PageRank

A search engine's effectiveness is determined by the importance of the results it returns. Text based ranking system used initially, which counted keyword occurrences in text files was flawed in this regard because it often failed to prioritise the most relevant results. That is the reason why modern

search engines display results to searches by placing results in a ranked order - with the results deemed "most important" at the top of the page.

PageRank, named after co-founder Larry Page, is an algorithm used by Google which attempts to determine the importance of a website. It is important to note that PageRank is not an algorithm exclusive to Google, however the name attribution to the google co-founder is due to Larry Page and Sergey Brin's significant progress at Stanford University from the initial concept of the Eigenvalue Problem.

The PageRank algorithm was influenced by previous multiple techniques including citation analysis and HyperSearch. The core idea is based on the principle that the importance of a web page can be determined by the number and quality of links pointing to it. This is to attempt to ensure the website pages which are "highly recommended", so one of the first few links displayed after a google search, are likely to be the most reputable and closest to the answer the user was searching for.

In mathematical terms, the problem can be described as finding the principal eigenvector of the link matrix of the web. The link matrix represents the web pages as nodes and the hyperlinks as directed edges.

Problems with criteria of determining importance of a page

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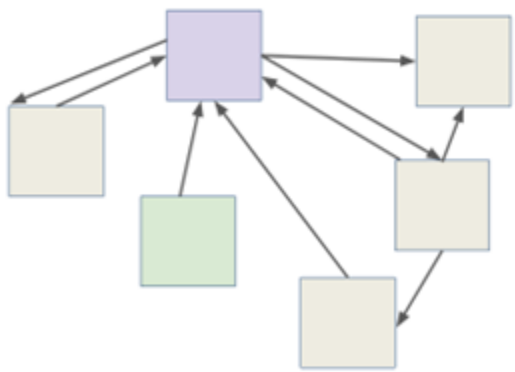
Pages on the internet are sorted into intricate webs of links depending on the user's search. Since pages are connected to each other by links, the algorithm assumes that if a page is linked to by many pages, it is probably more important than a webpage linked to by fewer pages.

Therefore, a page with the greatest number of incoming links is deemed most important by this criterion, and conversely a page with fewer incoming links is deemed less important. The following diagram is an example of a network of webpages with varying number of links between them. In this example, the purple page, with 4 incoming links would be deemed most important and the green page would be deemed least important.



However, if this criterion were to be used solely, it would be extremely easy to inflate artificially the importance of a webpage, with spoofing or other webpages being created with the sole purpose to provide an additional link and inflate the importance of a particular webpage for monetary or status gain. Moreover, if there were multiple groups of networks, only one network would be populated with a mark of importance.

Random Surfer Model

Given these problems it was evident that a more nuanced definition of 'importance' needed to be found in order to improve the effectiveness of page ranking. PageRank algorithm uses a more nuanced definition: the importance of a page is judged by the number of pages linking to it as well as the importance of those linking pages. This appears to lead to a paradox since the ability to calculate the importance of a page requires knowing another page's importance.

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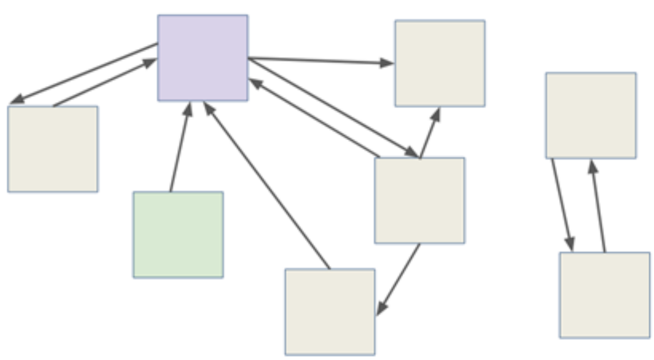
The random surfer model provides a basis for the PageRank algorithm and calculating an appropriate score for each page, to counter any chances that all links contribute to a page's authority signals. The model imagines a surfer who starts with a web page at random, and then randomly chooses links from that page (and subsequent pages) to follow. It also assumes that that the user will at some point lose interest and leave so the number of successive links is not infinite. Pages that have more links to them are more likely to be visited so they will have higher scores and because those pages are more likely to be visited the pages they link to are also more likely to be visited so a link from a more important page will matter more than a link from a less important page. The total score for each pages gives a measure for the relative importance of these pages represented as the amount of times the random surfer can be expected to land on a particular page in a network.

The inflation of a page's importance has been solved by the more nuanced definition of an important page. However, there is still one problem with this approach though and it is the fact that pages on the Internet might not all be connected to each other, as illustrated in the diagram below, therefore making it impossible for the random surfer to jump between the networks shown in the diagram below.



Damping Factor

The algorithm introduces a damping factor to avoid infinite loops of the random web surfer only ever visiting one set of pages on the web (the network on the left in the diagram) while completely ignoring the rest of the internet (the network on the right in the diagram) since none of the other pages are reachable via any of the links from the pages originally visited.

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Directly linked pages have a higher chance of being visited, with the standard damping factor for connected pages being 0.85 compared to 0.15 for unconnected pages. If the damping factor is 0.85 for example that means that 85% of the time our random surfer will follow a link from the page that are on before 15% of the time switching instead to a page on the internet chosen completely at random. This leads to eventually all pages on the internet being visited and their relative importance deemed by the percentage it has been visited. In the first few steps the random surfer takes the numbers are not particularly accurate as a lot is based just on random chance but with enough time the random surfer will continue to explore more and more and the numbers eventually converge to a stable PageRank value for each page and those values are then be used to determine what order search results should appear in with the more important pages appearing first.

In summary, PageRank transformed web search by leveraging mathematical principles to evaluate page importance and its profound impact illustrates the significance of advanced mathematical algorithms in organising and accessing information on the web.

2. Dijkstra's Algorithm use in Google Maps

Graph Theory is also used by Google Maps to find the most convenient way to go from one place to another: in fact, the Web Service can determine, among all the possible paths, the shortest way to reach the wanted destination thanks to Dijkstra's Algorithm.

2.1) Dijkstra's Algorithm

The aim of Dijkstra's Algorithm is to find the shortest path connecting two vertices of a weighted graph; the starting vertex is called “source” vertex.
This path equals the set of edges that minimizes the sum of the associated weights.
To use the algorithm in Google Maps, we must interpret the network of roads as a weighted graph: each crossroad and corner is a vertex of the graph and each road connecting them is an edge with an assigned value (as distance or time).

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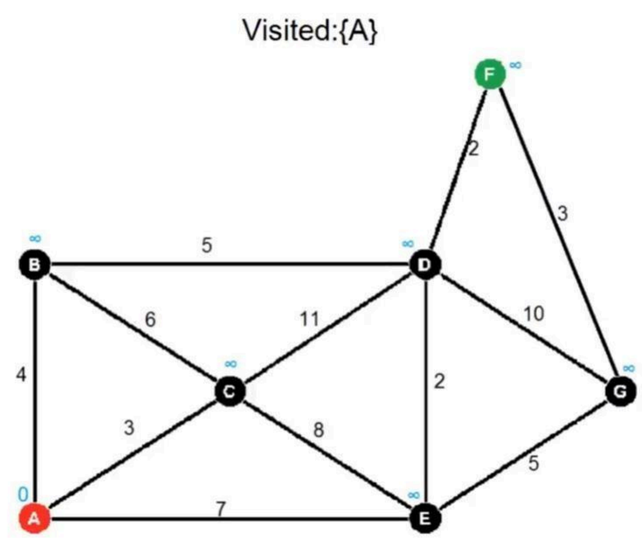
2.2) How the algorithm works

1) Select the starting vertex and the ending one.
2) Associate a cost zero to the starting vertex (because since we're already there, the distance and time needed to reach it is 0) and a starting value of infinite to all the other vertices (which means that a minimum estimated value to reach them hasn't been found yet).
3) From the starting vertex, assign a cost to all the adjacent vertices equal to the value of the path needed to reach them from the source.
4) Select the vertex with the lowest cost and mark it as visited: the shortest path to reach it from the source has been found. Keep track of the values assigned to the other vertices.
5) Repeat step 3 and 4 with the unvisited vertices adjacent to the last visited vertex. If there are two possible costs that can be assigned to one same vertex, choose the lower one (which is equal to the value of the minimum path).
6) Stop when the destination vertex is marked as visited.

The cost associated with the destination vertex corresponds to the distance (or time) needed to get there from the source, while the minimum path is given by the set of edges which weights have been summed to obtain that cost.

Example on a weighted undirected graph

1) After selecting the starting vertex (A) and the destination vertex (F), we associate a value of 0 to the source vertex and a value of infinite to all the other ones.



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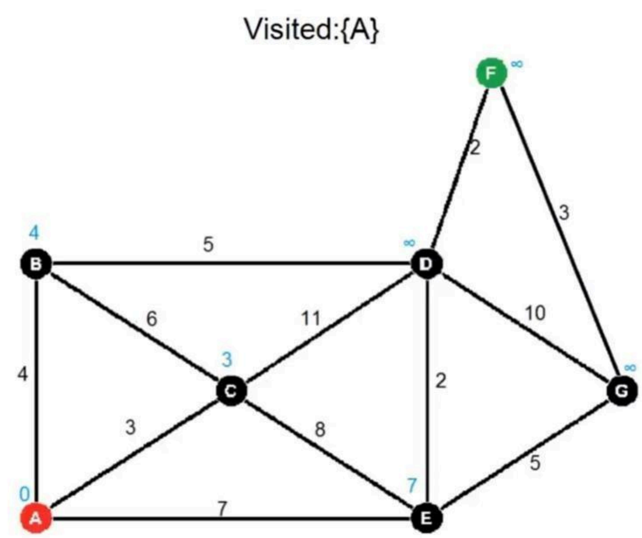
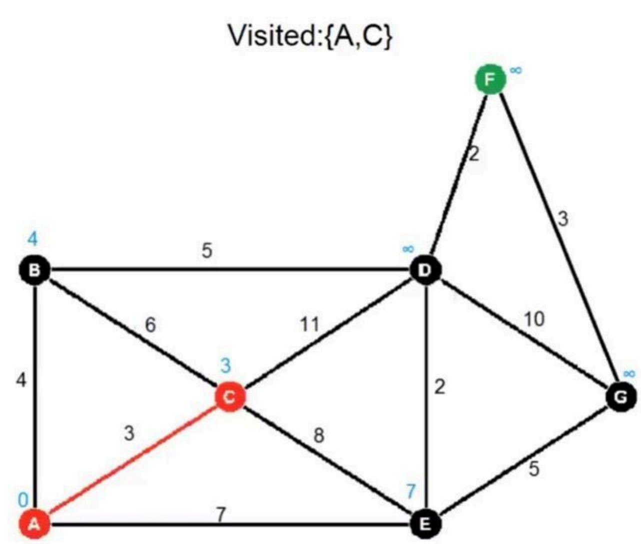
2) We assign a cost to all the vertices adjacent to A (B, C, E) equal to the weight of the edges connecting them to the source.



3) We select C and mark it as visited, since it's the vertex with the lowest cost associated.



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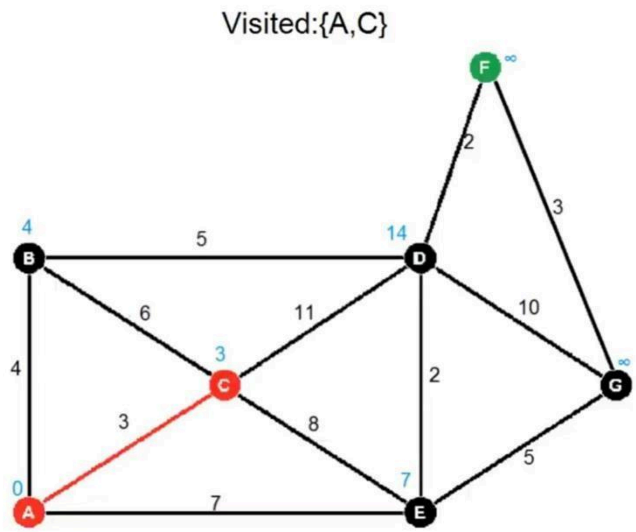
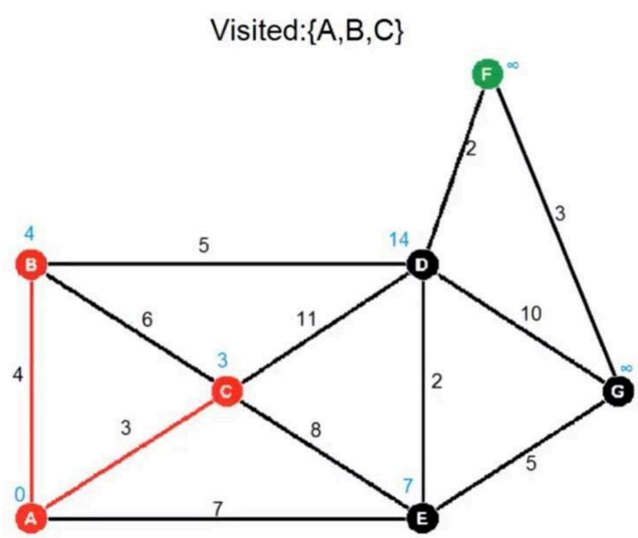
4) The unvisited vertices adjacent to C (the last visited vertex) are B, D, and E. We assign a cost of 14 to D, but we don't update the values of neither B nor E, since the new ones (9 and 11) would be higher than their actual cost.



5) We select B and mark it as visited, since it's the unvisited vertex with the lowest cost associated.



6) We select B and mark it as visited, since it's the unvisited vertex with the lowest cost associated.

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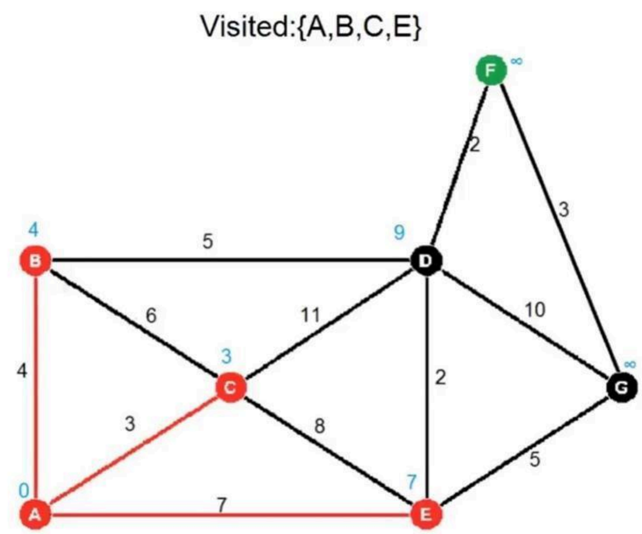
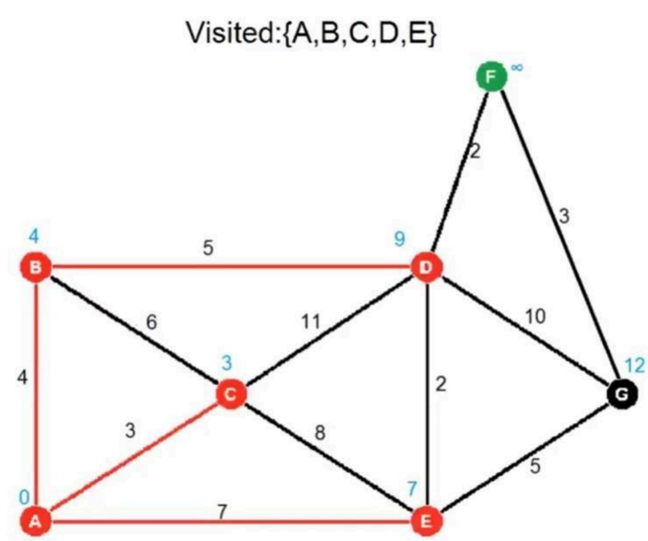


7) We assign a value of 12 to G, since the value of D remains 9. We then add D to the list of the visited vertices.



8) Since the new cost associated with G would be 19, which is higher than 12, we just assign a value of 11 to F. Then we mark F as visited and we can stop the algorithm:

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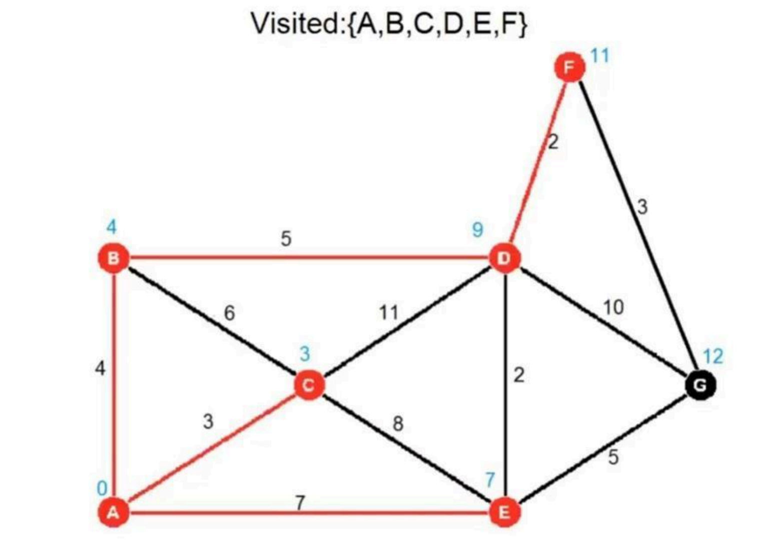
we have reached the destination vertex and found both the minimum value (11) and the shortest path (A-B-D-F) to go from the source to it.



3. Artificial Intelligence (AI) Algorithms

Introduction

This section of the paper will elaborate briefly on the concept of Artificial Intelligence (AI) and the algorithms within it. AI is revolutionizing the modern world. When used responsibly, AI can assist students in studying, enhance a user's experience online and digital assistants can quickly summarise an answer to a question, so you don't have to search through different websites on the internet for a simple question. As useful as AI can be, it also has its limits. The limitations and future possibilities will also be discussed in this section.

3.1) What is AI?

AI (artificial intelligence) is the science of making technology think like humans. AI solves problems, learns from data, and mimics human actions. The term was coined by John McCarthy in the 1950s, who believed that machines could one day simulate human intelligence. AI has many useful applications and is an evolving field of study.

3.2) What is an AI algorithm?

An algorithm is a set of instructions to be followed when solving calculations and other problems. An AI algorithm is a set of rules that can programme the computer to operate on its own. This allows the machine to learn, analyse data, complete tasks and make certain decisions based on its knowledge. Without an algorithm, AI would not exist.

3.3) The 4 types of AI algorithms

1. Supervised Learning - this is like a student learning in the presence of a teacher. Labelled data is presented and is used to learn and predict outcomes from other sets of data. Data scientists are present to correct errors and check accuracy.

2. Unsupervised Learning – Unlabeled data is presented and used to evaluate the relationships between different sets of data.

3. Both supervised and Unsupervised - both labelled and unlabeled data is given.

4. Reinforcement Learning – This is where AI learns by receiving feedback from the result of its action.

There are multiple different algorithms under these categories.

3.4) Uses of AI algorithms

AI can be applied to solve an array of inconveniences that can be experienced in daily life but can also have more significant applications.

• Search engines such as Google, Bing or Yahoo.

• Coding on platforms such as CoLab.

• Digital assistants like Alexa, Cortana and Siri.

• Helping students with work, for example Chat GPT.

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• Applications in robotics like self-driving cars etc.

3.5) Graph Theory in AI

Graph theory is applied in AI in a multitude of ways, such as:

• Social Network Analysis: Finding connections within social networks.

• Recommendations: Predicting a user's preferences by considering connections between users and searches.

• Knowledge Graphs: Presenting information in a way machines can understand and make inferences about.

• Molecular Modelling: Displaying molecules as graphs to predict properties and interactions.

• Route Selection: Finding the shortest and most efficient routes in logistics and transport.

3.6) The limitations of AI

AI holds significant promise for the future, but it also has a range of limitations. Some of these are:

• Limited Data Availability: AI systems rely on data. Insufficient, incorrect or biased data will hinder their performance. If data is biased, the responses given will not be accurate or assessed fairly.

• Resources: Training sophisticated AI models requires immense computational power and memory. This can be expensive, but this large use of energy can also have a negative effect on climate change.

• Security Vulnerabilities: AI systems can be vulnerable to attacks that manipulate their behavior and compromise security. Like any human using a computer can get hacked, AI is not immune. This can be extremely dangerous as it can affect the responses given by the computer, which should always be as reliable as possible.

• Privacy and Harmful uses of AI: As AI's popularity grows, regulations around privacy, safety, and liability need careful consideration. AI can generate images and videos which can be deceiving and threatening if used in a cruel way. Technology is easier to monitor than the people using it.

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3.7) The future of AI

The future of AI could hold many possibilities. Some of the main predictions are:

• Natural Language Processing (NLP): AI should continue to improve its language understanding. This would enable better chatbots, translation services, and generated responses.

• Computer Vision: This can entail enhanced images, detection of dangerous objects, and facial recognition which will impact fields such as healthcare and security.

• Services for human convenience: AI-powered machines and devices may be able to perform difficult tasks in manufacturing, logistics, and even domestic chores.

• Healthcare: AI could aid in the early detection of harmful illnesses, personalize treatments for individuals, and assist in the discovery of drugs.

3.8) Spectral Graph Theory and PageRank: Investigating the Influence of Algebraic Connectivity

Network analysis and optimization have become critical as networked systems become increasingly complex and important. Enhancing the efficiency and performance of networks requires understanding their structural properties. Algebraic connectivity, derived from the Laplacian matrix's spectral properties, is a key metric for evaluating and optimizing networks. Communication networks, smart grids, and mobile robotics all benefit from this metric, as it affects connectivity and performance (Martín-Hernández et al., 2014). The importance of algebraic connectivity has been highlighted in various contexts in recent research. For example, optimizing algebraic connectivity has improved the performance of communication networks in smart grids (Sydney et al., 2013) and enabled the synthesis of resilient networks with specific constraints (Nagarajan et al., 2014). Studies have shown that algebraic connectivity affects the convergence rate of consensus algorithms in asymmetric networks (Asadi et al., 2016). In interconnected networks, phase transitions have been observed with the addition of links among interdependent networks (Martin-Hernandez, 2013). The potential to increase algebraic connectivity without adding new links or nodes has also been explored, demonstrating the importance of network structure (Olfati-Saber, n.d.). Further, algebraic connectivity's

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role in cognitive radio ad-hoc networks and its comparison with other metrics like network criticality and betweenness centrality deepens its significance in network analysis (Abbagnale & Cuomo, 2010; Bigdeli et al., 2009; Deng, 2013). Despite these advancements, several gaps remain in the research on algebraic connectivity. For instance, while much is known about its application in various network types, less is understood about its behavior in emerging network structures or under dynamic conditions. The comparative effectiveness of algebraic connectivity relative to other network metrics in specific contexts remains debatable. Addressing these gaps is crucial for further advancing network optimization techniques and understanding their broader implications. This review aims to consolidate and evaluate the current research on algebraic connectivity, focusing on its applications, limitations, and potential for future advancements. By examining the cooperation between algebraic connectivity and other network metrics, the review seeks to provide an understanding of its role in network analysis and optimization.

Literature Review

Spectral Graph Theory, a branch of mathematics and computer science, has gained significant attention due to its applications in various fields such as computer science, physics, and biology. One key concept within Spectral Graph Theory is the algebraic connectivity of a graph, which is defined as the second smallest eigenvalue of the Laplacian matrix associated with the graph. This parameter serves as a measure of how well-connected a graph is, providing insights into its structural properties and behavior (Ghosh & Boyd, 2006). The algebraic connectivity plays a crucial role in understanding the strength and resilience of networks to node and link failures. Research by Jamakovic & Uhlig (2007) explores the relationship between algebraic connectivity and a graph's ability to withstand node and link failures, highlighting the importance of this parameter in assessing network reliability (Jamakovic & Uhlig, 2007). Moreover, MartíN-Hernández et al. (2014) explored the algebraic connectivity of interdependent networks, showcasing its significance in the context of network-of-networks (NoN) and the overall stability of interconnected systems (Martín-Hernández et al., 2014). Furthermore, the algebraic connectivity has been linked to consensus algorithms in multi-agent systems, where it influences the convergence rates of such algorithms. Algebraic connectivity affects the convergence rates of consensus algorithms, which are fundamental in multi-agent control and optimization techniques (Chen et al., 2021). This establishes a connection between increasing the algebraic connectivity of complex networks and enhancing their robustness to link and node failures, showcasing the practical implications of this

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parameter in network dynamics (Olfati-Saber, n.d.). In regard to graph theory, algebraic connectivity serves as a key metric for assessing the connectivity and structural properties of graphs. Algebraic connectivity is the second smallest eigenvalue of the graph Laplacian which highlights its role as a measure of graph connectivity (Ghosh & Boyd, 2006). Moreover, algebraic connectivity has been studied about various graph properties such as matching numbers, domination numbers, and pendant vertices. Lower bounds for the algebraic connectivity based on the matching number or edge covering several graphs showcase the cooperation between algebraic connectivity and other graph parameters (Xu, 2014). In the context of PageRank, an algorithm used for ranking web pages in search engine results, the algebraic connectivity has been linked to the heat kernel and personalized PageRank vectors. The Heat kernel of a graph is related to its PageRank which showcases a significance in tangent to PageRank in addressing fundamental challenges in large information networks leverage personalized PageRank vectors in a short-graph Fourier transform (Chung, 2007; Tepper & Sapiro, 2016).

Discussion

Algebraic connectivity is defined as the second smallest eigenvalue of the Laplacian matrix associated with a graph. It serves as a critical measure of a graph's connectivity and structural integrity (Ghosh & Boyd, 2006). Studies by Jamakovic & Uhlig (2007) emphasize the significance of algebraic connectivity in assessing network robustness to node and link failures. This highlights its importance in maintaining the resilience of complex networks (Jamakovic & Uhlig, 2007; Olfati-Saber, 2005). Algebraic connectivity is also closely linked to consensus algorithms in multi-agent systems. It impacts the convergence rates of these algorithms. Chen et al. (2021) discuss how increasing algebraic connectivity enhances these convergence rates. This demonstrates its practical implications in multi-agent control and optimization techniques. In graph theory, algebraic connectivity plays a crucial role in understanding the connectivity and structural properties of graphs. Ghosh and Boyd (2006) define algebraic connectivity as a fundamental metric for measuring graph connectivity. This highlights its significance in characterizing graph structures. Additionally, studies by Fan & Tan (2018), Xu (2014), and Lal et al. (2010) provide insights into how algebraic connectivity changes with modifications to graph structures. They offer lower bounds for this parameter based on graph properties such as matching numbers and pendant vertices. When applying algebraic connectivity to PageRank algorithms, Chung (2007) explores the relationship between the heat kernel of a graph and its PageRank. This demonstrates how PageRank leverages spectral graph theory concepts to address challenges in large information

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networks. Furthermore, Tepper & Sapiro (2016) showcase the integration of personalized PageRank vectors in algorithmic developments. This highlights the practical utility of Spectral Graph Theory in enhancing information retrieval and analysis processes. The literature also discusses optimizing algebraic connectivity within various network structures. Sydney et al. (2013) investigates the impact of maximizing algebraic connectivity on hierarchical communication networks in smart grids. They emphasize its role in enhancing network resilience and traffic characteristics. Similarly, Zhang et al. (2017) examine the algebraic connectivity of graphs with given stability numbers. They shed light on the relationship between stability numbers and algebraic connectivity in connected graphs. The reviewed literature reveals that algebraic connectivity is consistently emphasized across different applications. There are, however, varying interpretations of how algebraic connectivity affects specific outcomes, including network robustness and algorithm convergence. Chen et al. (2021) emphasizes the optimization benefits while Jamakovic & Uhlig (2007) focus on network resilience. The impact of algebraic connectivity on networks is multidimensional, spanning structural, functional, and operational aspects. When comparing different studies and viewpoints in Spectral Graph Theory and PageRank, it becomes evident that algebraic connectivity plays an important role in assessing network properties. Some studies focus on how strong and reliable a network is and how agreements are reached within it. Others look at the impact of these concepts on graph theory, which is the study of networks. For instance, Ghosh & Boyd (2006) and Fan & Tan (2018) lay down the theoretical groundwork, while Sydney et al. (2013) and Zhang et al. (2017) delve into real-world applications. Algebraic connectivity, a measure of how well-connected a network is, has proven to be adaptable and significant across various fields. Its diverse uses in different situations emphasize its flexibility and significance across disciplines. The research on Spectral Graph Theory and PageRank has far-reaching consequences. By grasping the impact of algebraic connectivity, both researchers and practitioners can create more resilient networks, develop effective consensus methods, and improve information retrieval systems. Incorporating algebraic connectivity into network analysis and improvement provides opportunities for boosting network efficiency, durability, and structural soundness.

Conclusion

This study shows how important algebraic connectivity is for making PageRank algorithms more efficient and reliable. PageRank algorithms are crucial for determining the ranking of web pages in search engine results. By exploring the relationship between

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algebraic connectivity and PageRank, this research demonstrates that improving graph connectivity can result in faster convergence rates and greater stability in large information networks. This finding has significant implications for designing and optimizing network structures, especially in areas where robust and efficient information retrieval is vital, such as computer science, telecommunications, and data science. The broader significance of these findings lies in the potential to develop more resilient and efficient networks, capable of withstanding failures and adapting to dynamic conditions. This study not only enhances our theoretical understanding of spectral graph theory and PageRank but also provides practical insights for improving the performance of real-world network systems.

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"Algebraic Connectivity Influences"

Author: Samuel J. Li

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Abstract

Based on spectral graph theory, this paper examines the role of algebraic connectivity in the PageRank Algorithm. A graph's algebraic connectivity is the second smallest eigenvalue of its graph Laplacian. The performance of PageRank is examined in terms of convergence speed, stability, and accuracy as a result of changes in algebraic connectivity. This study combines theoretical insights with practical experiments to investigate how graph connectivity affects ranking algorithms.

1. Introduction

Network analysis and optimization have become critical as networked systems become increasingly complex and important. Enhancing the efficiency and performance of networks requires understanding their structural properties. Algebraic connectivity, derived from the Laplacian matrix's spectral properties, is a key metric for evaluating and optimizing networks. Communication networks, smart grids, and mobile robotics all benefit from this metric, as it affects connectivity and performance (Martín-Hernández et al., 2014). The importance of algebraic connectivity has been highlighted in various contexts in recent research. For example, optimizing algebraic connectivity has improved the performance of communication networks in smart grids (Sydney et al., 2013) and enabled the synthesis of resilient networks with specific constraints (Nagarajan et al., 2014). Studies have shown that algebraic connectivity affects the convergence rate of consensus algorithms in asymmetric networks (Asadi et al., 2016). In interconnected networks, phase transitions have been observed with the addition of links among interdependent networks (Martin-Hernandez, 2013). The potential to increase algebraic connectivity without adding new links or nodes has also been explored, demonstrating the importance of network structure (Olfati-Saber, n.d.). Further, algebraic connectivity's role in cognitive radio ad-hoc networks and its comparison with other metrics like network criticality and betweenness centrality deepens its significance in network analysis (Abbagnale & Cuomo, 2010; Bigdeli et al., 2009; Deng, 2013). Despite these advancements, several gaps remain in the research on algebraic connectivity. For instance, while much is known about its application in various network types, less is understood about its behavior in emerging network structures or under dynamic conditions. The comparative effectiveness of algebraic connectivity relative to other network metrics in specific contexts remains debatable. Addressing these gaps is crucial for further advancing network optimization techniques and understanding their broader

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implications. This review aims to consolidate and evaluate the current research on algebraic connectivity, focusing on its applications, limitations, and potential for future advancements. By examining the cooperation between algebraic connectivity and other network metrics, the review seeks to provide an understanding of its role in network analysis and optimization.

Spectral Graph Theory, a branch of mathematics and computer science, has gained significant attention due to its applications in various fields such as computer science, physics, and biology. One key concept within Spectral Graph Theory is the algebraic connectivity of a graph, which is defined as the second smallest eigenvalue of the Laplacian matrix associated with the graph. This parameter serves as a measure of how well-connected a graph is, providing insights into its structural properties and behavior (Ghosh & Boyd, 2006). The algebraic connectivity plays a crucial role in understanding the strength and resilience of networks to node and link failures. Research by Jamakovic & Uhlig (2007) explores the relationship between algebraic connectivity and a graph's ability to withstand node and link failures, highlighting the importance of this parameter in assessing network reliability (Jamakovic & Uhlig, 2007). Moreover, MartíN-Hernández et al. (2014) explored the algebraic connectivity of interdependent networks, showcasing its significance in the context of network-of-networks (NoN) and the overall stability of interconnected systems (Martín-Hernández et al., 2014). Furthermore, the algebraic connectivity has been linked to consensus algorithms in multi-agent systems, where it influences the convergence rates of such algorithms. Algebraic connectivity affects the convergence rates of consensus algorithms, which are fundamental in multi-agent control and optimization techniques (Chen et al., 2021). This establishes a connection between increasing the algebraic connectivity of complex networks and enhancing their robustness to link and node failures, showcasing the practical implications of this parameter in network dynamics (Olfati-Saber, n.d.). In regards to graph theory, algebraic connectivity serves as a key metric for assessing the connectivity and structural properties of graphs. Algebraic connectivity is the second smallest eigenvalue of the graph Laplacian which highlights its role as a measure of graph connectivity (Ghosh & Boyd, 2006). Moreover, algebraic connectivity has been studied about various graph properties such as matching numbers, domination numbers, and pendant vertices. Lower bounds for the algebraic connectivity based on the matching number or edge covering several graphs showcase the cooperation between algebraic connectivity and other graph parameters (Xu, 2014). In the context of PageRank, an algorithm used for ranking web pages in search engine results, the algebraic connectivity has been linked to the heat kernel and personalized PageRank vectors. The Heat kernel of a graph is related to its

PageRank which showcases a significance in tangent to PageRank in addressing fundamental challenges in large information networks leverage personalized PageRank vectors in a short-graph Fourier transform (Chung, 2007; Tepper & Sapiro, 2016).

2. Discussion

Algebraic connectivity is defined as the second smallest eigenvalue of the Laplacian matrix associated with a graph. It serves as a critical measure of a graph's connectivity and structural integrity (Ghosh & Boyd, 2006). Studies by Jamakovic & Uhlig (2007) emphasize the significance of algebraic connectivity in assessing network robustness to node and link failures. This highlights its importance in maintaining the resilience of complex networks (Jamakovic & Uhlig, 2007; Olfati-Saber, 2005). Algebraic connectivity is also closely linked to consensus algorithms in multi-agent systems. It impacts the convergence rates of these algorithms. Chen et al. (2021) discuss how increasing algebraic connectivity enhances these convergence rates. This demonstrates its practical implications in multi-agent control and optimization techniques. In graph theory, algebraic connectivity plays a crucial role in understanding the connectivity and structural properties of graphs. Ghosh and Boyd (2006) define algebraic connectivity as a fundamental metric for measuring graph connectivity. This highlights its significance in characterizing graph structures. Additionally, studies by Fan & Tan (2018), Xu (2014), and Lal et al. (2010) provide insights into how algebraic connectivity changes with modifications to graph structures. They offer lower bounds for this parameter based on graph properties such as matching numbers and pendant vertices. When applying algebraic connectivity to PageRank algorithms, Chung (2007) explores the relationship between the heat kernel of a graph and its PageRank. This demonstrates how PageRank leverages spectral graph theory concepts to address challenges in large information networks. Furthermore, Tepper & Sapiro (2016) showcase the integration of personalized PageRank vectors in algorithmic developments. This highlights the practical utility of Spectral Graph Theory in enhancing information retrieval and analysis processes. The literature also discusses optimizing algebraic connectivity within various network structures. Sydney et al. (2013) investigate the impact of maximizing algebraic connectivity on hierarchical communication networks in smart grids. They emphasize its role in enhancing network resilience and traffic characteristics. Similarly, Zhang et al. (2017) examine the algebraic connectivity of graphs with given stability numbers. They shed light on the relationship between stability numbers and algebraic connectivity in connected graphs. The reviewed literature reveals that algebraic connectivity is

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consistently emphasized across different applications. There are, however, varying interpretations of how algebraic connectivity affects specific outcomes, including network robustness and algorithm convergence. Chen et al. (2021) emphasize the optimization benefits while Jamakovic & Uhlig (2007) focus on network resilience. The impact of algebraic connectivity on networks is multidimensional, spanning structural, functional, and operational aspects. When comparing different studies and viewpoints in Spectral Graph Theory and PageRank, it becomes evident that algebraic connectivity plays an important role in assessing network properties. Some studies focus on how strong and reliable a network is and how agreements are reached within it. Others look at the impact of these concepts on graph theory, which is the study of networks. For instance, Ghosh & Boyd (2006) and Fan & Tan (2018) lay down the theoretical groundwork, while Sydney et al. (2013) and Zhang et al. (2017) delve into real-world applications. Algebraic connectivity, a measure of how well-connected a network is, has proven to be adaptable and significant across various fields. Its diverse uses in different situations emphasize its flexibility and significance across disciplines. The research on Spectral Graph Theory and PageRank has far-reaching consequences. By grasping the impact of algebraic connectivity, both researchers and practitioners can create more resilient networks, develop effective consensus methods, and improve information retrieval systems. Incorporating algebraic connectivity into network analysis and improvement provides opportunities for boosting network efficiency, durability, and structural soundness.

3. Conclusion

This study shows how important algebraic connectivity is for making PageRank algorithms more efficient and reliable. PageRank algorithms are crucial for determining the ranking of web pages in search engine results. By exploring the relationship between algebraic connectivity and PageRank, this research demonstrates that improving graph connectivity can result in faster convergence rates and greater stability in large information networks. This finding has significant implications for designing and optimizing network structures, especially in areas where robust and efficient information retrieval is vital, such as computer science, telecommunications, and data science. The broader significance of these findings lies in the potential to develop more resilient and efficient networks, capable of withstanding failures and adapting to dynamic conditions. This study not only enhances our theoretical understanding of spectral graph theory and PageRank but also provides practical insights for improving the performance of real-world network systems.

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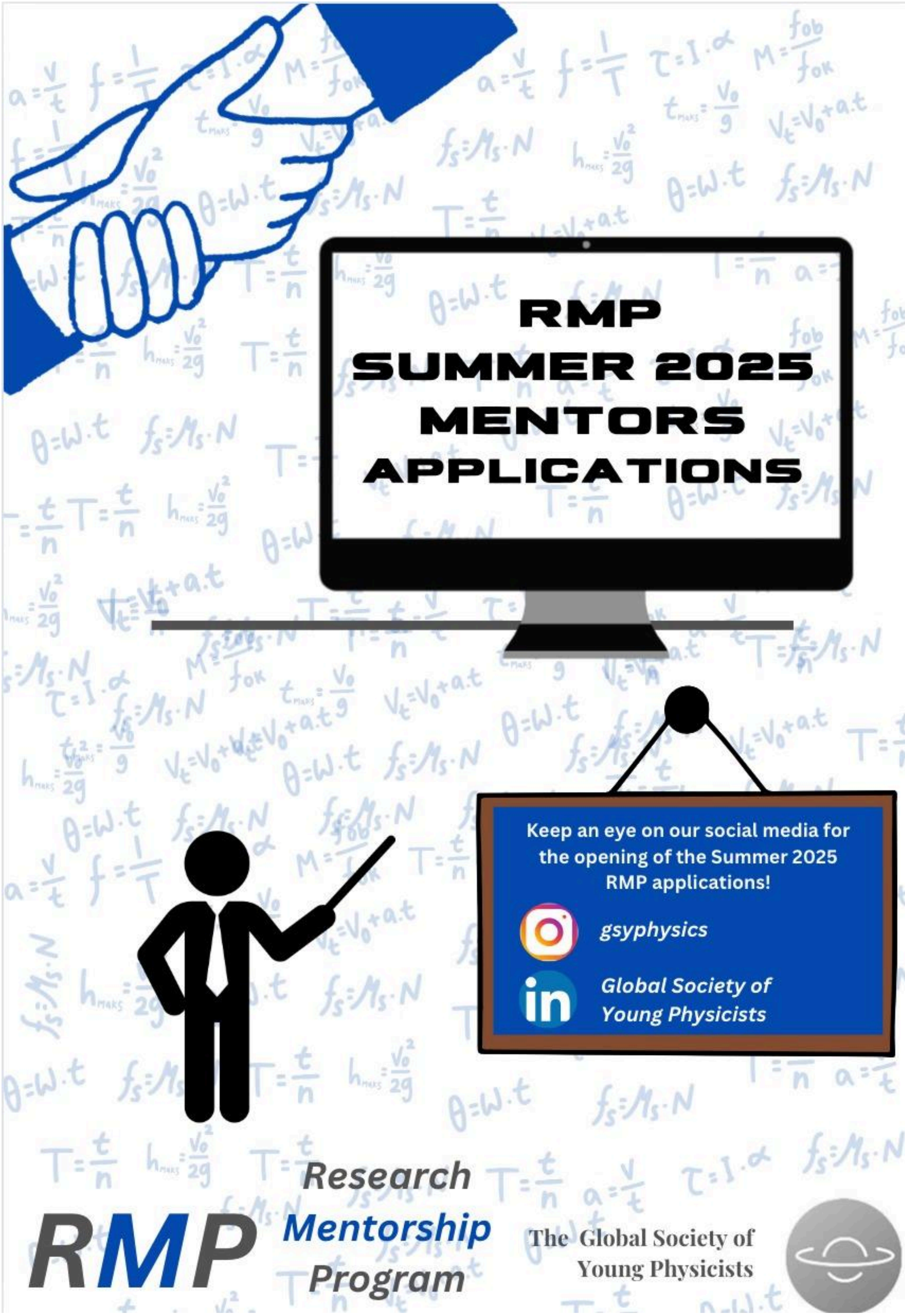

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Arya Lal Gonullu, Chief Executive of GSYP and Editor of the August issue of *The Starry Messenger*, named the science magazine after Galileo Galilei's groundbreaking work, *Sidereus Nuncius* (The Starry Messenger). In this historic book, Galileo had unveiled three revolutionary discoveries he made with it: Sunspots, the Moons of Jupiter and that the Moon is not a perfect sphere. Gonullu hopes that physics will soon experience a new “Starry Messenger” breakthrough through the new rising stars of the field.

Magazine Cover and Poster Design: Chiara Allegri, the Head of Marketing at GSYP & The Starry Messenger August Issue Deputy Editor

Research Mentors of RMP 2024

Quantum Mechanics	**Arya Lal Gonullu**
Mathematics	**Ethan Lee**
Cosmology	**Toby Bhanji** **Huu Duc Lu**
Experimental & Engineering Physics	**Piyamini Tenahandi**

This first issue of *The Starry Messenger* is dedicated to Selmin Dedeli.

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